A model of resource allocation during plant growth

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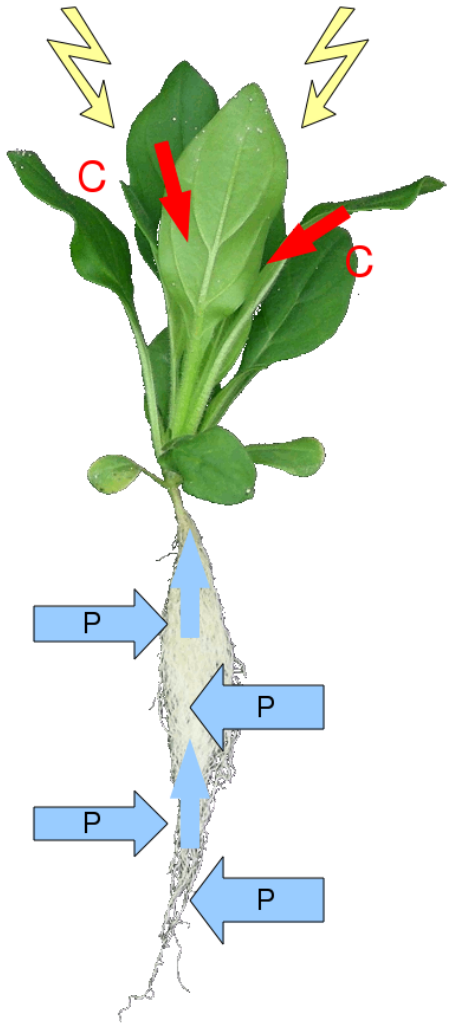
Introduction



Introduction: previous models

1. John H. M. Thornley (1972,1990,1991,1994,1997):
   Main differences with our model: continuous light, no respiration, no starch, no soil, the transport.
2. R.C. Dewar (1993):
   based on Thornley's model, added the effect of water potential
   predictions are not significantly different from Thornley.

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Schema of our model

2 compartments:

- the leaf compartment: source of sugar.
- the root compartment: source of phosphate.

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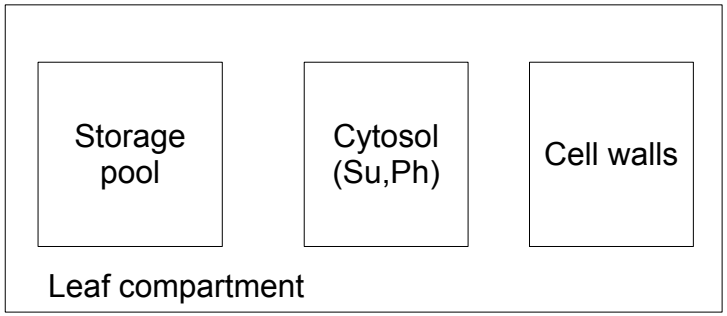

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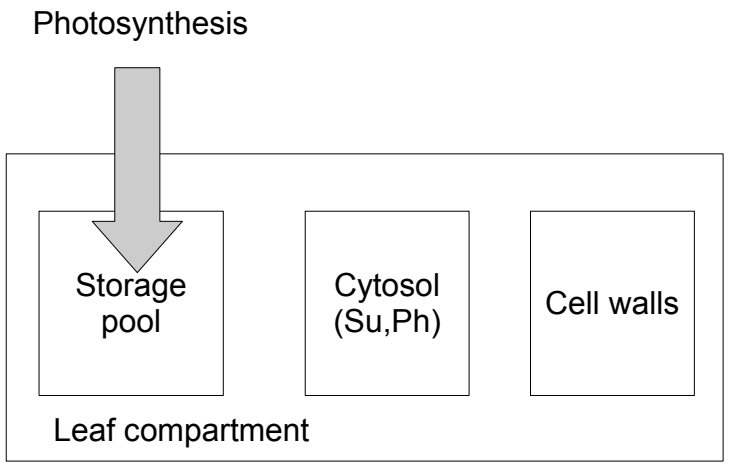

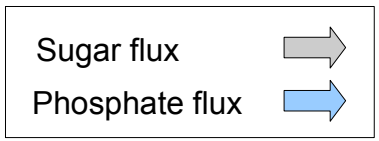

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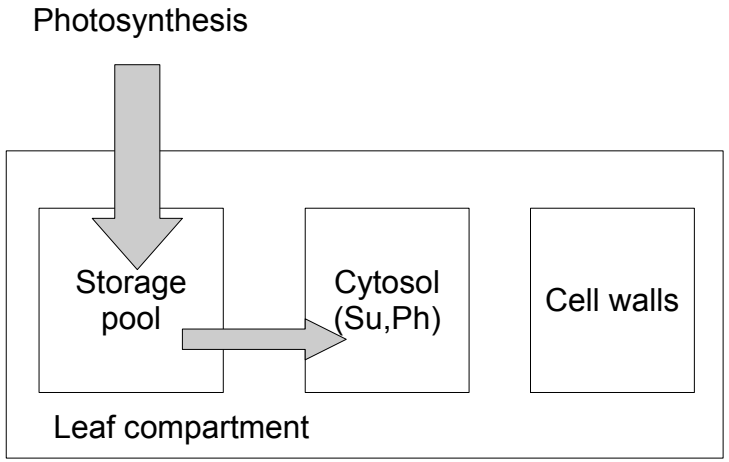

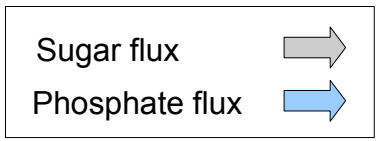

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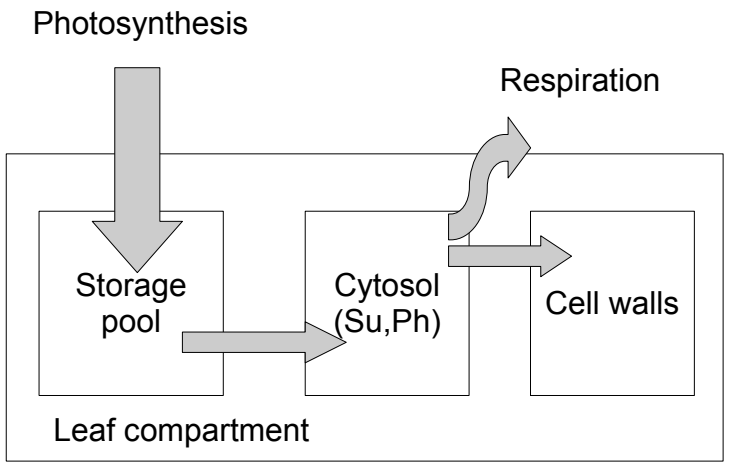

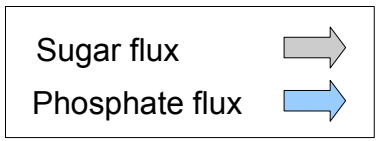

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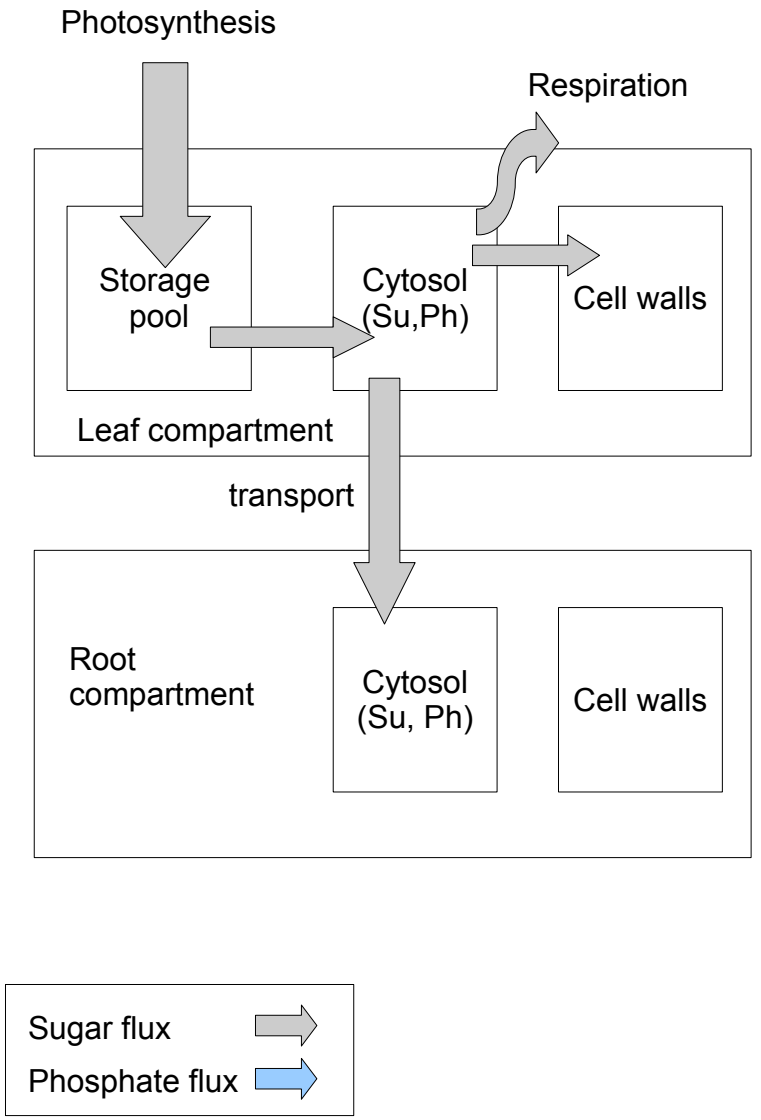

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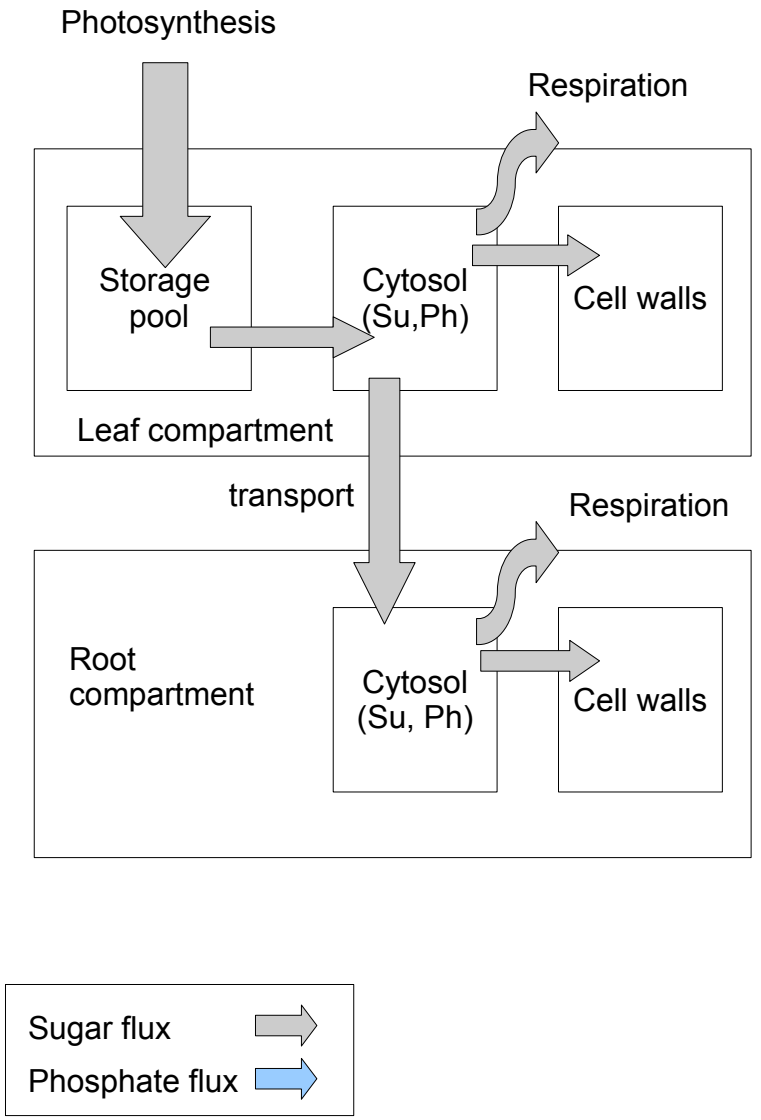

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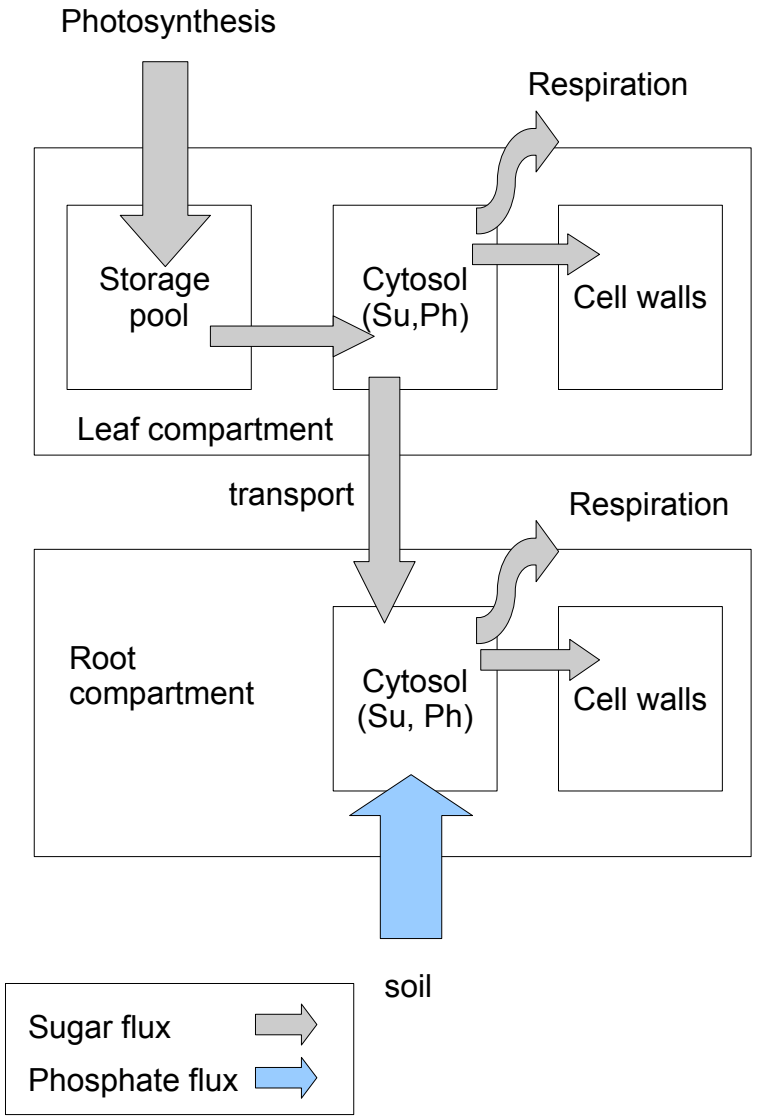

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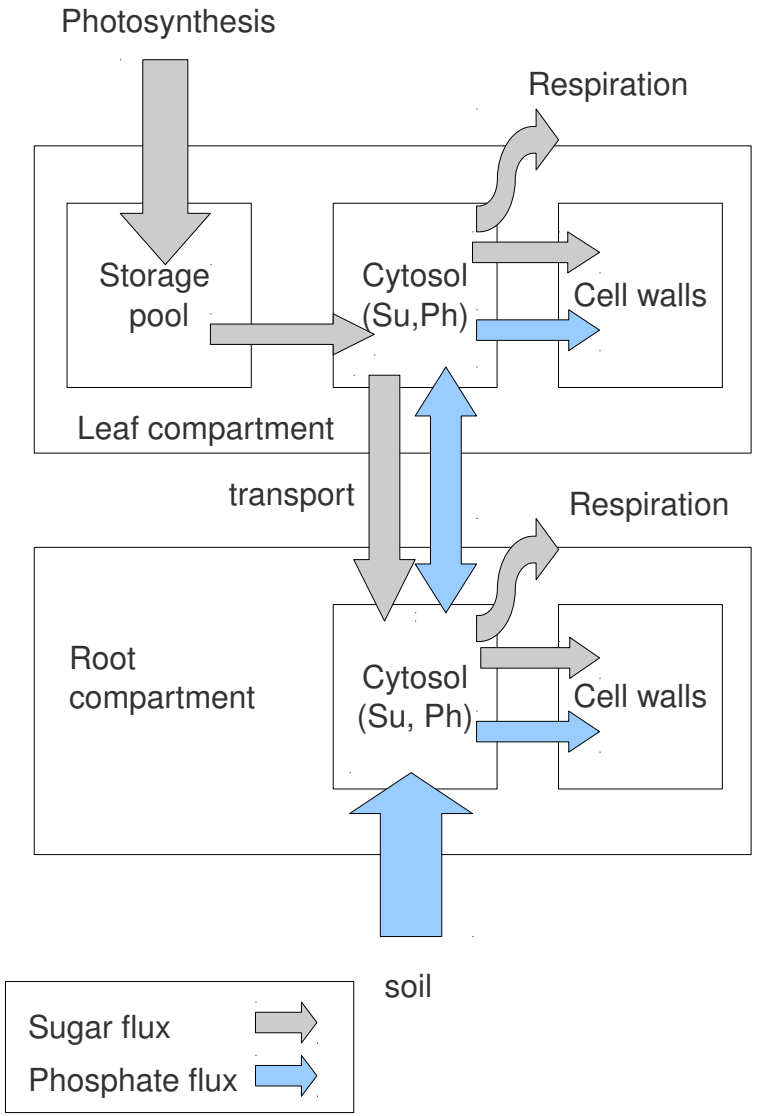
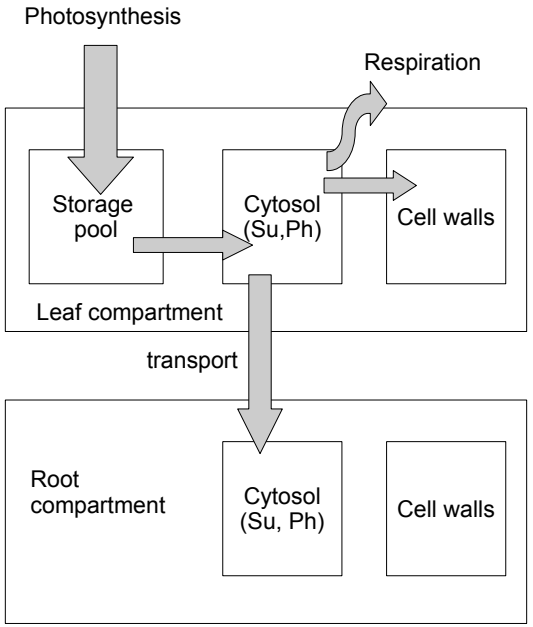
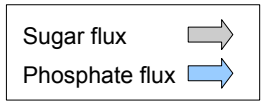


The dynamic of the sugar in the plant

$$\dot{Q}^{\ell}_{st}(t) = + \text{photosynthesis} - \text{destocking}$$

$$\dot{Q}^{\ell}_{su}(t) = + \text{destocking} - \text{growth} - \text{respiration} - \text{flux to the root comp.}$$





Example: Sugar flux to the root compartment

$$n(t)\, C^{\ell}_{su}\, (C^{\ell}_{su} - C^{r}_{su}) \frac{d\pi RTx^4}{8L(t)\eta}$$

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$$\begin{aligned}
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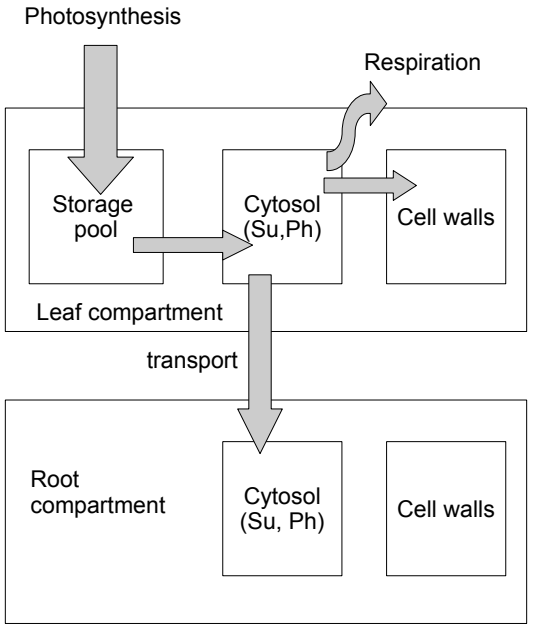

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The storage

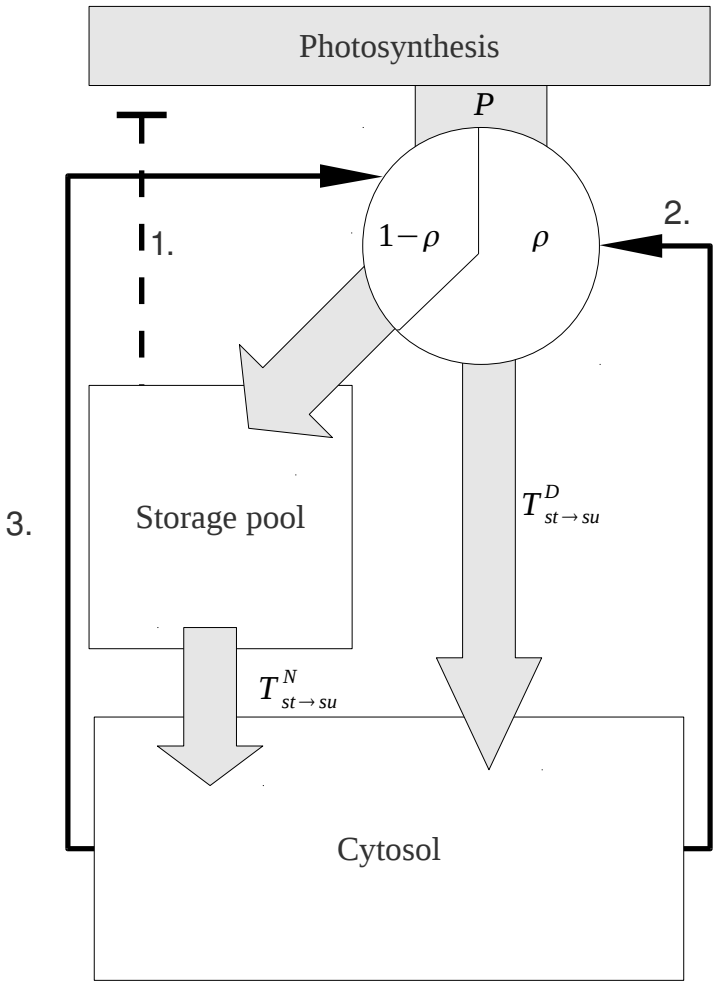
1. Feedback – if:

$C^l_{st}(t) > C^l_{st,max}$

3. Feedback + if:

$C^l_{su}(t) < C^l_{su,t}$

and $D(t) = 0$



2. Feedback + if:

$C^l_{su}(t) < C^l_{su,t}$

and $D(t) = 1$

The storage

During the night:

$$T^{N}_{st\to su}(t) := \frac{Q^{\ell}_{st}(t)}{L_{night}(t)} S^{-}_{41}(C^{\ell}_{su,t}, C^{\ell}_{su}(t))$$

where $L_{night}(t)$ is the expected lenght of the night.

Parameters

$\approx$ 80 parameters

2 types

- with a biological meaning.
  $\rightarrow$ maximal growth rate, respiration rate...
  $\rightarrow$ ESTIMATION METHODS: experiments, literature.
- Function coefficients .
  $\rightarrow$ phosphate uptake depend on

$$\frac{C_{ph}^{soil}(t)}{m + C_{ph}^{soil}(t)}$$

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Parameter estimation (Type 1): based on biological meaning

Model of the photosynthetic leaf surface $S^{\ell}_{photo}(t)$:

- $\frac{V^{\ell}(t)}{d(\mathcal{I})}$ if $S^{\ell}_{photo}(t) < S_c$
  with $d(\mathcal{I}) = d_{min} + (d_{max} - d_{min})\frac{\mathcal{I}}{a+\mathcal{I}}$ the leaf thickness
- $\alpha_1 + \alpha_2 (V^{\ell}(t))^{2/3}$ otherwise

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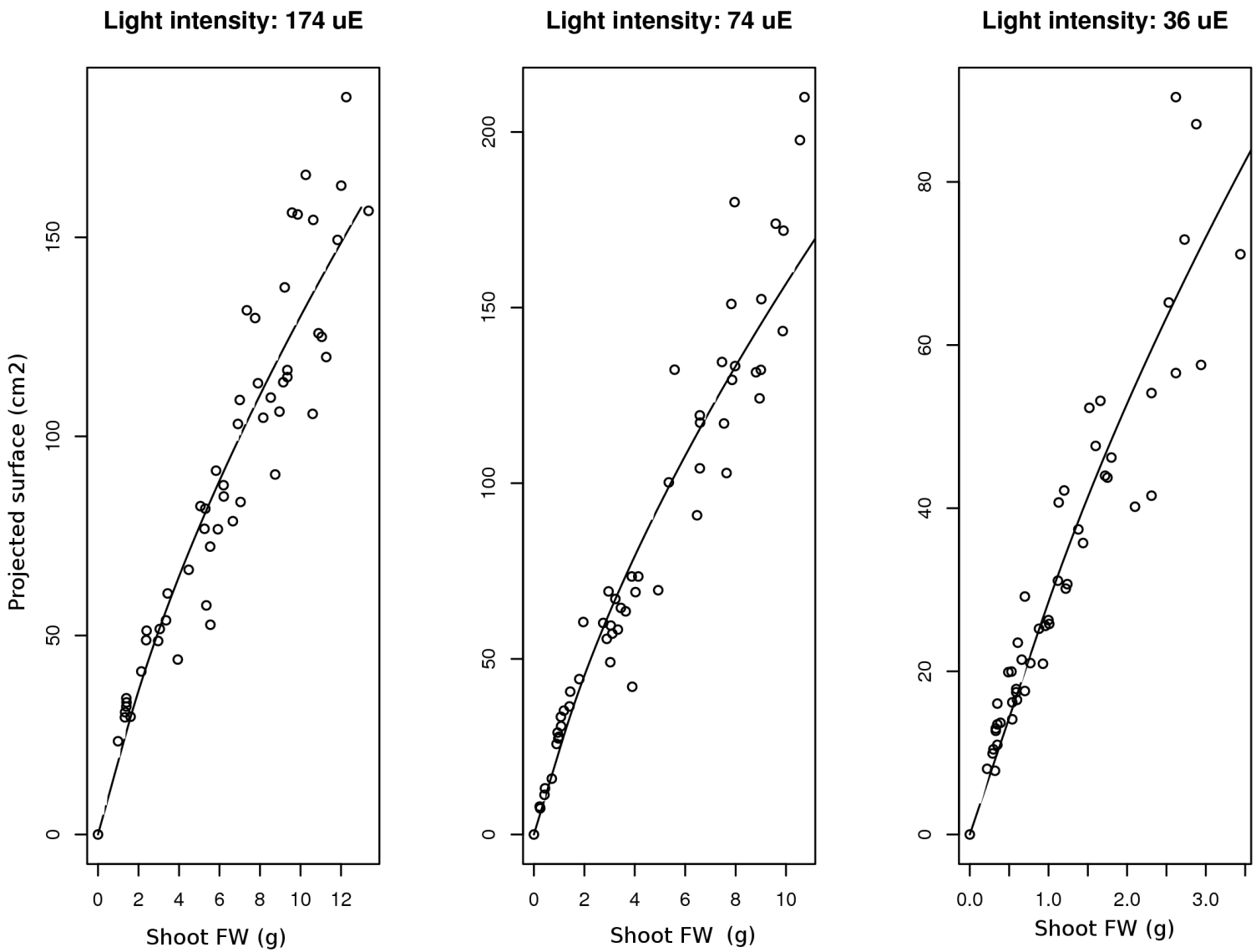
Parameters:

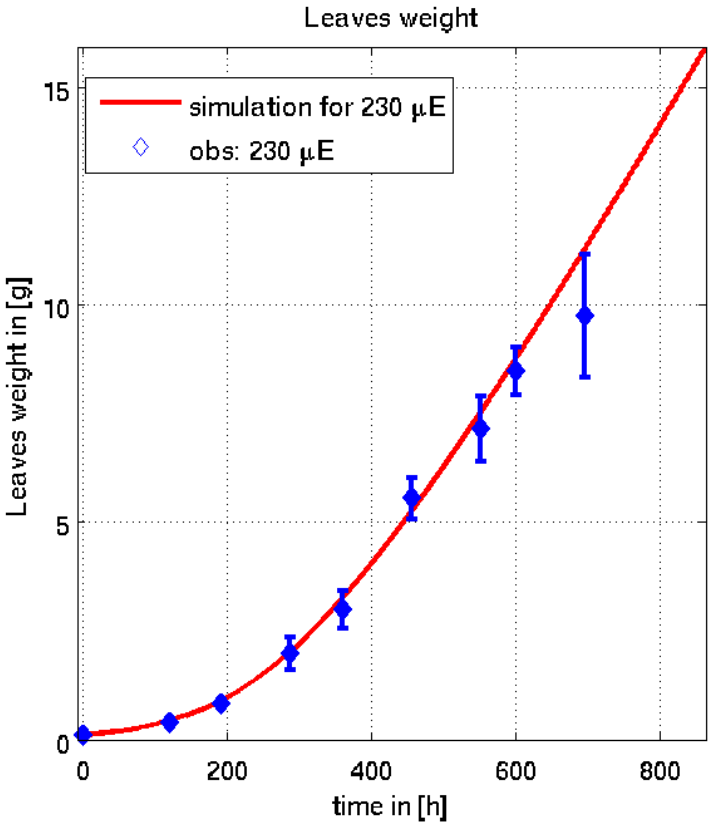
$$S_c, \alpha_1, \alpha_2, d_{min}, d_{max} \text{ and } a$$

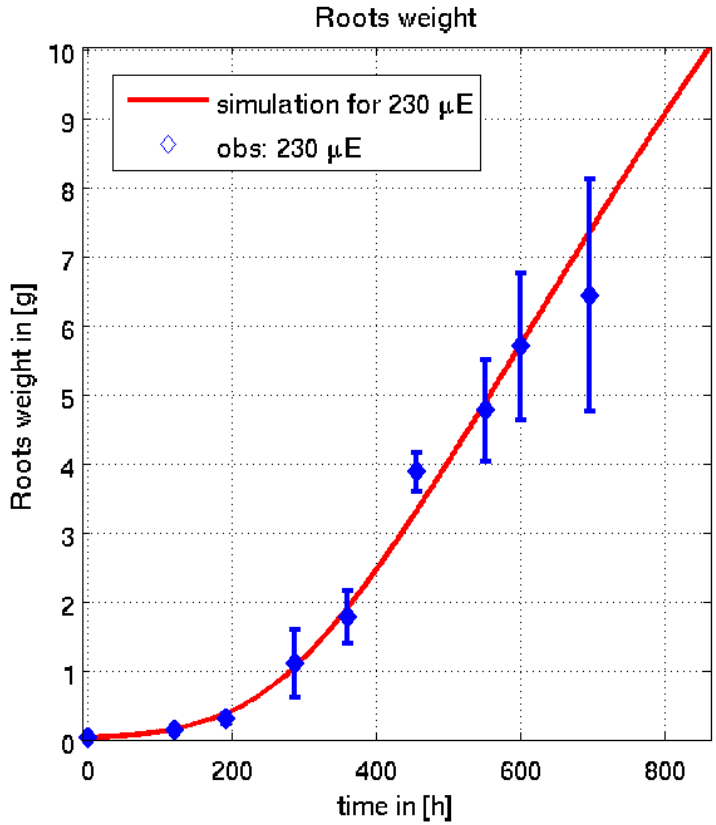
Parameter estimation (Type 1): based on biological meaning (2)



Parameter estimation (Type 2): based on model fit

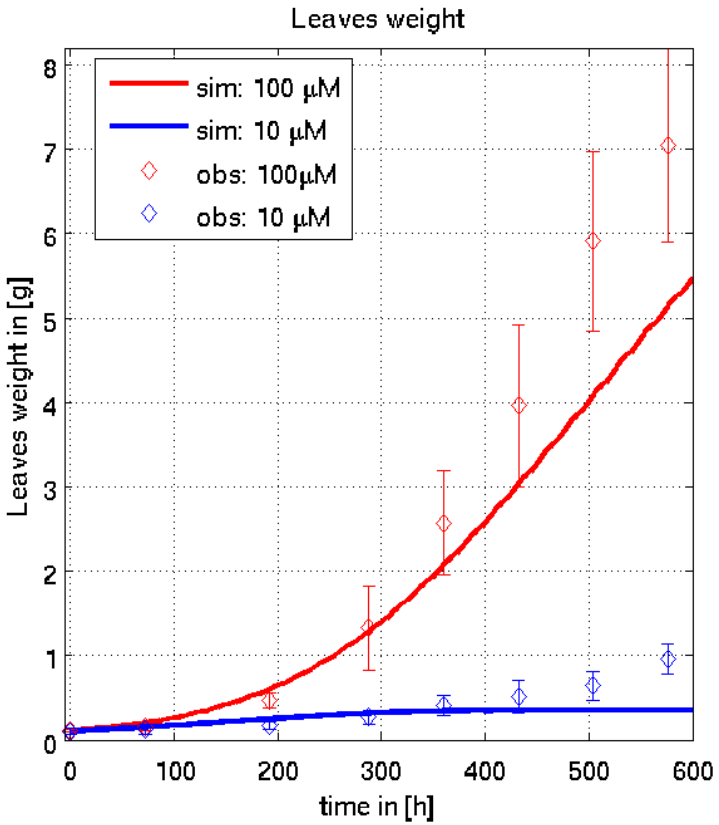
Leaves and roots growth under high light and high phosphate concentration in the soil:

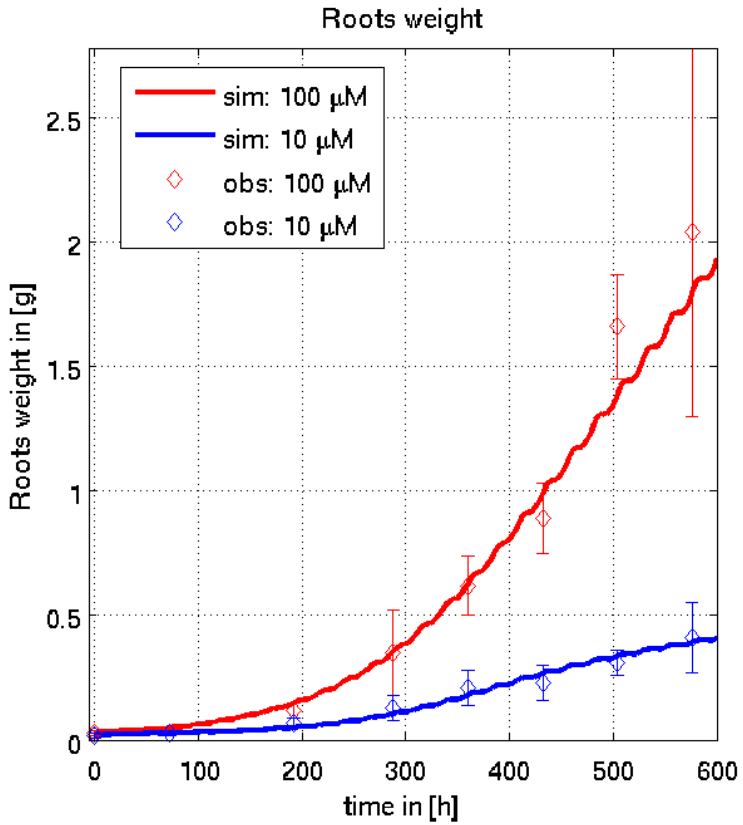




Parameter estimation (Type 2): based on model fit (3)

Leaves and roots growth under high light (122 uE)





Simulations: validation of the model (3)

Leaves and roots growth under high light (144 uE) and different phosphate concentrations in the soil