Task 3 (10 Cr):

Given is the following function:

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 7x - 5$$

a) Find all x values where the function have $f(x)$ local extrema and classify them as minima or maxima.

b) Find where the function is increasing/decreasing and all x values of inflection points.

Solution:

a)

$$f'(x) = x^2 - 8x + 7$$

$$x^2 - 8x + 7 = 0$$

$$x_{1,2} = \frac{8 \pm \sqrt{64 - 4 \cdot 1 \cdot 7}}{2} = \frac{8 \pm \sqrt{36}}{2}$$

$$f''(x) = 2x - 8$$

$$x_1 = 1; f''(x_1) = 2 \cdot 1 - 8 = -6 < 0 - maximum$$

$$x_2 = 7; f''(x_2) = 2 \cdot 7 - 8 = 6 > 0 - minimum$$

b)

$$f'(x) = (x-7)(x-1)$$

$$-\infty < x < 1 \;: f'(x) > 0: \; increasing$$

$$1 < x < 7 \;: \; f'(x) < 0: decreasing$$

$$x > 7 : f'(x) > 0 \; increasing$$

$$f''(x) = 0: \quad 2x - 8 = 0 \quad x = 4 \qquad f'''(x) = 2 \neq 0$$

$x = 4$: inflection point

Task 4 (10 Cr.):

Compute the total area between the function

$$f(x) = 6x^2 + 6x - 12$$

the x-axis and the lines $x_1 = 0$ and $x_2 = 2$.

Solution:

The roots of the function:

$$6x^2 + 6x - 12 = 0$$

$$x_{1,2} = \frac{-6 \pm \sqrt{36 + 4 \cdot 6 \cdot 12}}{12} = \frac{-6 \pm \sqrt{324}}{12}$$

$$x_1 = -2; x_2 = 1$$

$$f(x) = 6x^2 + 6x - 12 = 6(x+2)(x-1)$$

$x < -2$: function positive

$-2 < x < 1$: function negative

$x > 1$: function positive

$$Area = \left|\int_0^1 (6x^2 + 6x - 12)\, dx\right| + \int_1^2 (6x^2 + 6x - 12) dx =$$

$$= |[2x^3 + 3x^2 - 12x]_0^1| + [2x^3 + 3x^2 - 12x]_1^2 =$$

$$= |2 + 3 - 12| + (2 \cdot 2^3 + 3 \cdot 2^2 - 2 \cdot 12) - (2 + 3 - 12) = 7 + 4 - (-7)$$

$$= 18$$

Task 7 (7 Cr):

On one farm the weight of 100 cows was measured. The mean body weight was $\bar{x} = 720$ kg and the variance of sample was $s_x^2 = 400\ kg$. The body weight is approximately normal distributed.

a) Compute a 95% confidence interval for the mean weight.

b) Formulate the null hypotheses and answer the question, if the measured average weight of 100 cows is higher, than 700 kg ($\alpha = 0.05$).

Solution

a)

The 95% confidence interval for the mean weight:
Limits of the 95% confidence inteval:

$$lower\ limit: \overline{x} - z_{1-\alpha/2} \cdot \sqrt{\frac{s_x^2}{n}} = 720 - 1.96 \cdot 2 = 716.08$$

$$upper\ limit: \overline{x} + z_{1-\alpha/2} \cdot \sqrt{\frac{s_x^2}{n}} = 720 + 1.96 \cdot 2 = 723.92$$

b)

$H_0: \mu \leq 700$

$H_1: \mu > 700$

$$z_{Test} = \frac{\bar{x} - \mu}{\sqrt{\frac{s_x^2}{n}}} = \frac{720 - 700}{\sqrt{\frac{400}{100}}} = \frac{20}{2} = 10$$

$z_{Tab} = 1.64$

$z_{Test} > z_{Tab} \Rightarrow H_0$ is rejected

The average weight of 100 cows $\bar{x} = 720$ is significantly higher, than 700 kg.

Task 8 (3 Cr.):

Investigated was the relationship between the *height* of apple trees in m $(\boldsymbol{X})$ and the *yield* in kg $(\boldsymbol{Y})$.

The estimated regression equation was as follows:

$$\hat{y} = -0.5 + 1.4x$$

and the estimate of coefficient of correlation $r = 0.96$.

a) What yield do you expect by the **Height** = 3m?

b) What fraction of variability of $\boldsymbol{Y}$ can be explained by $\boldsymbol{X}$? Give your answer in words.

Solution:

a)

$$\hat{y} = -0.5 + 1.4 \cdot 3 = 3.7\ kg$$

b)

$$R^2 = 0.96^2 = 0.92$$

92% of variability of $\boldsymbol{Y}$ can be explained by $\boldsymbol{X}$