Computer Science and Mathematics

Test exam from Summer Term 2013 (also for Summer Term 2014)

Solutions for Tasks 1, 2, 5, 6

<u>Task 1</u> $\vec{a} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} -16 \\ 0 \\ -10 \end{bmatrix}$

(a) $2\vec{a} + \vec{b} = \begin{bmatrix} 2\cdot 1 + 6 \\ 2\cdot(-2)+4 \\ 2\cdot 2 + 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 5 \end{bmatrix}$ (b) $\vec{a}\cdot\vec{b} = 1\cdot 6 + (-2)\cdot 4 + 2\cdot 1 = 0$

(c) $\sphericalangle(\vec{a},\vec{b}) = \arccos \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|} = \arccos 0 = 90°$ (or $\frac{\pi}{2}$, in radians)

(d) Solution with the determinant:

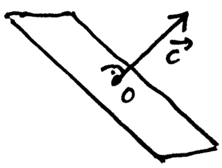
$$\begin{vmatrix} 1 & 6 & -16 \\ -2 & 4 & 0 \\ 2 & 1 & -10 \end{vmatrix} = 1\cdot 4\cdot(-10) + 0 + (-2)\cdot 1\cdot(-16) - 2\cdot 4\cdot(-16) - 0 - (-10)\cdot(-2)\cdot 6$$
$$= -40 \quad +32 \quad +128 \quad -120$$
$$= 0 \quad \Rightarrow \text{ the column vectors } \vec{a}, \vec{b}, \vec{c} \text{ are lin. dependent;}$$

or solve the system $m_1\cdot\vec{a} + m_2\cdot\vec{b} + m_3\cdot\vec{c} = \vec{0}$ and show that it has more than the trivial solution $m_1 = 0, m_2 = 0, m_3 = 0$;

or find such a solution by trial-and-error, e.g.,

$$(+4)\cdot\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + (+2)\cdot\begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} + (+1)\cdot\begin{bmatrix} -16 \\ 0 \\ -10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} .$$

(e) $\vec{x}$ fulfills $\vec{c}\cdot\vec{x} = 0$ if and only if $\vec{x}$ is orthogonal to the given vector $\vec{c}$ $\Rightarrow$ the set of points is the plane through O which is orthogonal to $\vec{c}$.



<u>Task 2</u> $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$.

(a) $\operatorname{rank}(A) = 2$, because $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ are linearly independent; or: because $\begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4 \neq 0$.

(b) $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$. (See extra figure.)

(c) f mirrors every vector at the principal diagonal and then doubles its length.

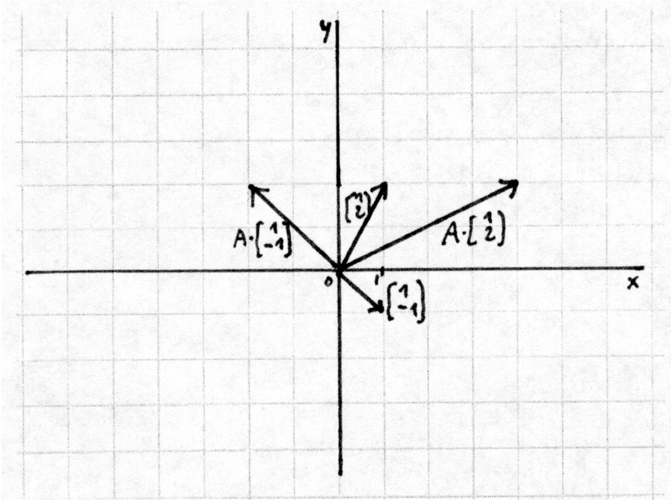
(d) $A^2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

(e) $\det A = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = 0 - 4 = -4$

(f) $A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

(g) $\det(A - \lambda E) = \begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} = \lambda^2 - 4 = (\lambda-2)\cdot(\lambda+2) \overset{!}{=} 0 \Leftrightarrow \lambda_1 = -2,\ \lambda_2 = +2$

task 2 (b), graphical part:



Task 5

i goes through the positions of array x, starting with 0.

As soon as the corresponding entry of x (x[i]) is equal to a, b gets the value "false" and the while-loop stops.

i is then returned, i.e., the result is the position of the first entry equal to a in array x.

If no "a" is found in x, b remains true, and the value −1 is returned.

Task 6

(a) $63_{10} = 111111_2$

(b) $2A5_{16} = 2\cdot 256 + 10\cdot 16 + 5\cdot 1 = 677_{10}$

(c) $84_{10} = 01010100_2$ [8 bits]

$\Rightarrow 10101011$

$+ \quad 1$

$10101100 \quad \Rightarrow -84 \mathrel{\hat{=}} 10101100$

(d) $x = 0.2222\ldots_3$

$3x = 2.2222\ldots_3 = 2 + x \quad \Rightarrow 3x - x = 2$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$