Topics for the next 4 weeks: The second part of Mathematics

1. **Sequences**
2. **Limit of a Function**
3. **Differentiation**
4. **Partial Derivatives**
5. **Curve Discussion and Extreme Value Problem**
6. **Integration**

Sources:

http://www.youtube.com/playlist?list=PLF5E22224459D23D9

http://www.youtube.com/playlist?list=PLDE28CF08BD313B2A

http://www.pages.drexel.edu/~gln22/Lecture%20Notes%20on%20Calculus.htm

Sequences

Consider the infinite "list" of terms:

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}$$ ← formula for n^{th}term

In general:

$$a_1, a_2, a_3, \dots a_n, \qquad a_n - \ n^{th} term$$

Or as a function

$$a(1), a(2), a(3), a(4), \dots, a(n)$$

Definition:

- A sequence is a **function** whose Domain is $\mathbb{N} = \{1, 2, 3, 4, \dots, n\}$

- In practice, we usually refer to the infinite list of its Range values as the sequence:

$$a(1), a(2), a(3), a(4), \dots, a(n)$$

Notation:

$$\{a_n\} = \{a_1, a_2, a_3, \dots a_n\}$$

or

$$\{a_n\}_{n=1}^{\infty} = \{a_1, a_2, a_3, \dots a_n\}$$

A sequence is often given by the n^{th} term formula (also called **general term**):

Exercise 1 : Write the first 5 terms:

$$\{a_n\} = \frac{1}{n}, n = 1,2,3,4, \ldots$$

Solution 1:

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots$$

Exercise 1 : Write the first 5 terms:

$$a_n = \frac{1}{2^n}, n = 1,2,3,4, \dots$$

Solution 2:

$$\frac{1}{2^0}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^5}, \dots \dots ..$$

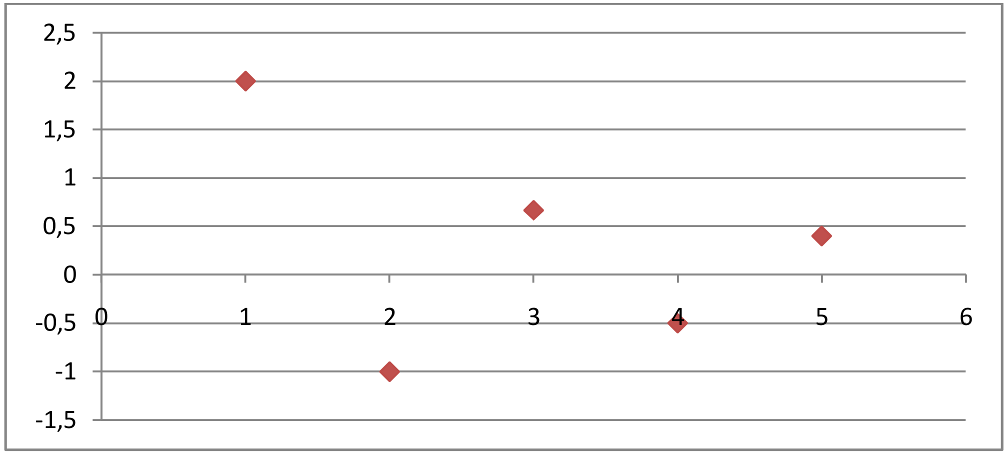
Exercise 3: Write the first 5 terms:

$$\{b_n\} = \left\{(-1)^{n-1} \cdot \left(\frac{2}{n}\right)\right\}$$

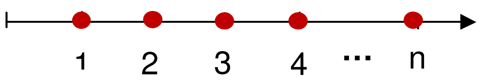
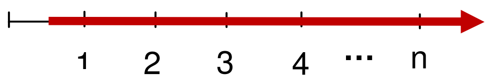
Solution 3:

$$\{b_n\} = \left\{\frac{2}{1}, \frac{-2}{2}, \frac{2}{3}, \frac{-2}{4}, \frac{2}{5}, \ldots\right\}$$

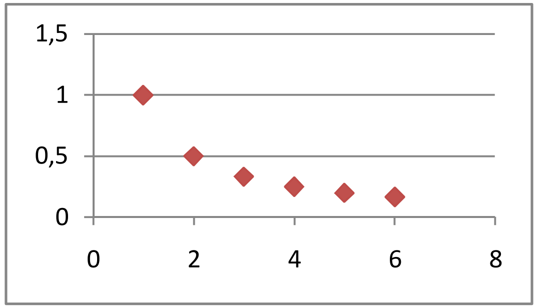
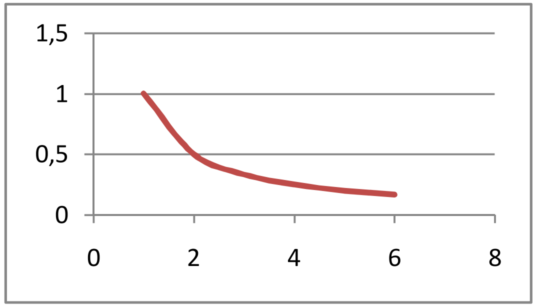
Graph 3:



Distinctions between sequences and functions:

<table>
<tr><th>Sequence</th><th>Function</th></tr>
<tr><td>$a(n) = a_n, n \in \mathbb{N}$</td><td>$f(x), x \in \mathbb{R}$</td></tr>
<tr><td>Dom a: Discrete Numbers: 1, 2, 3…</td><td>Dom f: Continuous Intervals</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Range $a(n)$
$a = \{a_1, a_2, a_3, \dots a_n, \dots \dots\}$</td><td>Range $f(x)$= some Interval</td></tr>
</table>

Example:

Sequence	Function
$a_n = \frac{1}{n}, n \in \mathbb{N}$	$f(x) = \frac{1}{x}, x \in [1, \infty)$
$Rana = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$	$Ran\ f(x) = (0,1]$
	
Graph consisting discrete discontinuous <u>Dots</u>	Graph Continuous Curve

Warning: You cannot determine a sequence from only a finite number of terms!

Example:

a_1	a_2	a_3	a_4	$\dots$	a_n
1	3	9	27	$\dots$	3^{n-1}
1	3	9	19	$\dots$	$1 + 2(n-1)^2$
1	3	9	11	$\dots$	$8n + \frac{12}{n} - 19$

Often a sequence is given by a recursive formula

- Stating its 1st term (s), then
- Writing a formula for the n^{th} term involving some preceding terms. This is called **a recursive formula**

Example:

$$a_1 = 1$$

$$\underbrace{a_n}_{subsequent} = 4 \cdot \underbrace{a_{n-1}}_{previous} \quad : recursive\ formula$$

Solution:

$$a_1 = 1$$

$$a_2 = 4 \cdot a_1 = 4 \cdot 1 = 4$$

$$a_3 = 4 \cdot a_2 = 4 \cdot 4 = 16$$

$$a_4 = 4 \cdot a_3 = 4 \cdot 16 = 64$$

Limits of sequences

Some sequences “approach” a number as you move out of the sequence: e.g. the sequence

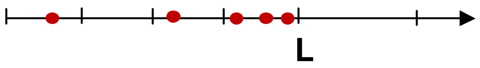
$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$$

“approaches” 0

A Sequence

$$\{a_n\}_{n=1}^{\infty} = \{a_1, a_2, a_3, \ldots\}$$

is said to **converge** to a number L (called a limit of the sequence) if any interval around L (however small) contains all the terms of a sequence beyond some point



In this case we write:

$$L = \lim_{n\to\infty} a_n$$

If no such number exists we say that the sequence diverges.

Example 1

$$\{3^n\}_{n=1}^{\infty}$$

$$\{3^n\}_{n=1}^{\infty} = \{3, 3^2, 3^3, \ldots\}$$

clearly diverges

Example 2

$$\left\{\left(\frac{1}{2}\right)^n\right\}_{n=1}^{\infty}$$

$$\left\{\left(\frac{1}{2}\right)^n\right\}_{n=1}^{\infty} = \left\{\frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \ldots\right\}$$

converges to 0

Example 3

$$\{(1)^n\}_{n=1}^{\infty}$$

$$\{(1)^n\}_{n=1}^{\infty} = \{1, 1, 1, \dots\}$$

converges to 1

In general:

$$\{x^n\}_{n=1}^{\infty}$$

converges to 0, if $-1 < x < 1$, converges to 1 if $x = 1$ and diverges for every other value of x