Computer Science and Mathematics: additional exercises

Additional exercises – Solutions

Task 1 (sets)

(a) $B = \{1; 3; 5; 7; 9\}$

(b) $A \cup B = \{1; 2; 3; 5; 7; 9\}$

(c) $A \cap B = \{1; 3\}$

(d) $\mathcal{P}(A) = \{\emptyset; \{1\}; \{2\}; \{3\}; \{1;2\}; \{1;3\}, \{2;3\}; \{1;2;3\}\}$

(e) $A \times C = \{(1;0); (1;1); (2;0); (2;1); (3;0); (3;1)\}$

(f) $C \times C \times C = \{(0;0;0); (0;0;1); (0;1;0); (0;1;1); (1;0;0); (1;0;1); (1;1;0); (1;1;1)\}$

Task 2

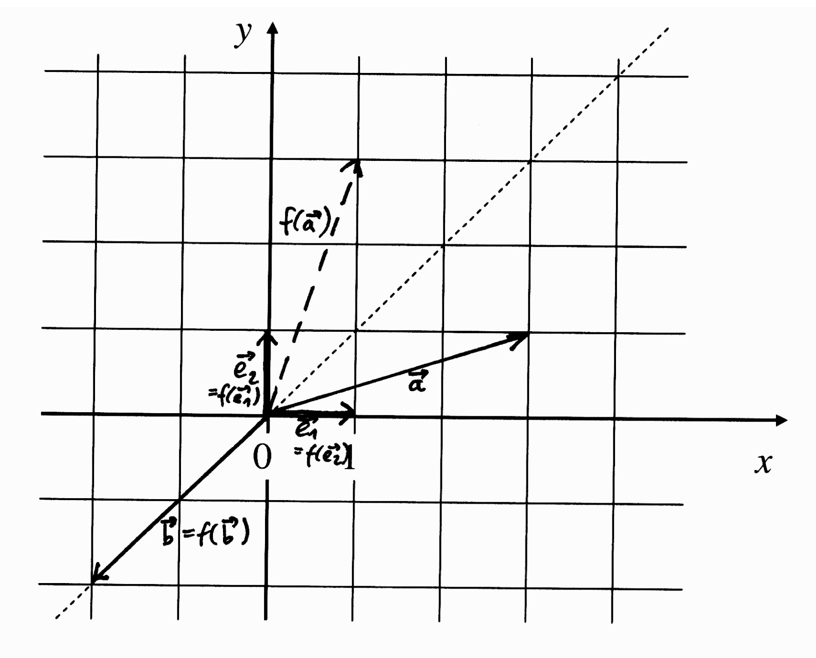
(a) (see extra figure)

$f(\vec{e}_1) = \vec{e}_2$, $f(\vec{e}_2) = \vec{e}_1$, $f(\vec{b}) = \vec{b}$

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = E$ (unit matrix)

(d) Applying the mirror transformation twice has the same effect as doing nothing (= applying the unit matrix)



Task 3

(a) $\vec{AB} = B - A = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}, \quad \vec{BC} = C - B = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{AC} = C - A = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}$

(b) $\|\vec{AB}\| = \sqrt{3^2 + (-2)^2 + 0^2} = \sqrt{13} \; (\approx 3.606)$

$\|\vec{BC}\| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2} \; (\approx 1.414)$

$\|\vec{AC}\| = \sqrt{3^2 + (-1)^2 + (-1)^2} = \sqrt{11} \; (\approx 3.317)$

(c) $\cos \measuredangle(\vec{AB}, \vec{AC}) = \dfrac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\| \cdot \|\vec{AC}\|} = \dfrac{3 \cdot 3 + (-2) \cdot (-1) + 0 \cdot (-1)}{\sqrt{13} \cdot \sqrt{11}} = \dfrac{11}{\sqrt{13} \cdot \sqrt{11}} = \dfrac{\sqrt{11}}{\sqrt{13}}$

(≈ 0.920)

(d) area (triangle) $= \frac{1}{2}$ area (parallelogram)

$$= \frac{1}{2} \| \vec{AB} \times \vec{AC} \| = \frac{1}{2} \left\| \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} \times \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix} \right\|$$

$$= \frac{1}{2} \left\| \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \right\| = \frac{1}{2} \sqrt{4+9+9} = \frac{1}{2} \sqrt{22} \; (\approx 2.345)$$

Task 4

(a) $\begin{vmatrix} 1 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 7 & 4 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 1 \\ 7 & 4 \end{vmatrix} + 0 + 0 = 8 - 7 = 1$

$\begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{vmatrix} = 2 \cdot 3 \cdot 5 = 30$

(b) $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 7 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 12 & 15 \\ 0 & 6 & 5 \\ 0 & 21 & 20 \end{bmatrix}$

(c)
$$\begin{array}{ccc|ccc|l}
1 & 4 & 3 & 1 & 0 & 0 & -2 \\
0 & 2 & 1 & 0 & 1 & 0 & :2 \\
0 & 7 & 4 & 0 & 0 & 1 & \\
\hline
1 & 0 & 1 & 1 & -2 & 0 & \\
0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & -7 \\
0 & 7 & 4 & 0 & 0 & 1 & \\
\hline
1 & 0 & 1 & 1 & -2 & 0 & -2 \\
0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & -1 \\
0 & 0 & \frac{1}{2} & 0 & -\frac{7}{2} & 1 & \cdot 2 \\
\hline
1 & 0 & 0 & 1 & 5 & -2 & \\
0 & 1 & 0 & 0 & 4 & -1 & \\
0 & 0 & 1 & 0 & -7 & 2 &
\end{array}$$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 5 & -2 \\ 0 & 4 & -1 \\ 0 & -7 & 2 \end{bmatrix}.$

Task 5

Matrix of the system:

$$\underbrace{\underbrace{\left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 17 & -3 & 0 \\ 0 & 2 & 5 & 22 & 5 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & 6 & 8 \end{array}\right.}_{A}\left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right]}_{A_{ext}} \begin{array}{l} \\ \\ \left.\right) -2 \\ \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 17 & -3 & 0 \\ 0 & 2 & 5 & 22 & 5 \\ 0 & 0 & 1 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \text{upper tri-angular} \\ \\ \\ \leftarrow \text{zero row} \end{array}$$

$$\Downarrow$$

$$\text{rank}(A) = \text{rank}(A_{ext}) = 3$$

$$< \text{nb. of unknowns} = 4$$

$$\Downarrow$$

the system has infinitely many solutions
(according to Frobenius' theorem)

Task 6

(a)
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 200 \\ 320 \\ 80 \end{bmatrix} \Leftrightarrow \begin{cases} x_1 + x_2 + x_3 = 200 & (1) \\ x_1 + 2x_2 + 3x_3 = 320 & (2) \\ x_2 + x_3 = 80 & (3) \end{cases}$$

(b) Eq. (3) $\Rightarrow x_3 = 80 - x_2$

$\hookrightarrow (1) \Rightarrow x_1 + x_2 + (80 - x_2) = 200 \Rightarrow x_1 = 120$

$\hookrightarrow (2) \Rightarrow 120 + 2x_2 + 3(80 - x_2) = 320 \Rightarrow -x_2 = 320 - 120 - 240 = -40$

$\Rightarrow x_2 = 40$

$\Rightarrow x_3 = 80 - 40 = 40$

(c) The system has a single solution $\Rightarrow \text{rank}(A) = n = 3 \Rightarrow \det(A) \neq 0$.

Task 7

Given are the functions:

$$f(x) = 5x^3 - \frac{15}{2}x^2 - 30x + 50$$

$$g(x) = 2 - \frac{1}{4}x^3$$

$$h(x) = 1 - 2x^3$$

(a) Determine the limits values

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \frac{5x^3 - 7.5x^2 - 30x + 50}{2 - 0.25x^3}$$

Deg(top)=Deg(bottom):

$$\lim_{x\to\infty} f(x) = \frac{leading\ coefficient\ of\ top}{leading\ coefficient\ of\ bottom} = -\frac{5}{0.25} = -20$$

$$\lim_{x\to 0}\frac{f(x)}{g(x)} = \frac{5x^3 - 7.5x^2 - 30x + 50}{2 - 0.25x^3} = \frac{50}{2} = 25$$

$$\lim_{x\to 2}\frac{f(x)}{g(x)} = \frac{5x^3 - 7.5x^2 - 30x + 50}{2 - 0.25x^3}$$

$$f(2) = 5 \cdot 8 - 7.5 \cdot 4 - 30 \cdot 2 + 50 = 40 - 30 - 60 + 50 = 0$$

$$g(2) = 2 - 0.25 \cdot 8 = 2 - 2 = 0$$

$\lim_{x\to 2}\frac{f(x)}{g(x)}$ has the form $\frac{0}{0} \to$ L'Hôpital Rule

$$\lim_{x\to 2}\frac{f(x)}{g(x)} = \frac{5x^3 - 7.5x^2 - 30x + 50}{2 - 0.25x^3} = \frac{15x^2 - 15x - 30}{2 - 0.75x^2} = \frac{0}{-1} = 0$$

(b) Determine the positions of the local extrema of f : where does this function reach a minimum, where a maximum?

$$f(x) = 5x^3 - \frac{15}{2}x^2 - 30x + 50$$

$$f'(x) = 15x^2 - 15x - 30$$

Critical points:

$$15x^2 - 15x - 30 = 0$$

$$x_{1,2} = \frac{15 \pm \sqrt{15^2 - 4 \cdot 15 \cdot (-30)}}{-2 \cdot 15} = \frac{15 \pm 45}{-30}$$

$$x_1 = -2;\ x_2 = 1$$

$$f''(x) = 30x - 15$$

$$x_1 = -2;\ f''(-2) = 30 \cdot (-2) - 15 = -75 < 0 \ \rightarrow \textit{local maximum}$$

$$x_2 = 1;\ f''(1) = 30 \cdot 1 - 15 = 15 < 0 \ \rightarrow \textit{local minimum}$$

(c) Draw the function *h* : flipped parabola, with top point at (0; 1).

- Prove that *h* is not injective.

"*h* injective" would mean: for all *a*, *b*: $a \neq b \Rightarrow h(a) \neq h(b)$.

But we have (e.g.): $h(-1) = 1 - 2\cdot(-1)^2 = -1 = 1 - 2\cdot 1^2 = h(1)$.

So *h* cannot be injective.

(d) Calculate $h(g(x))$ Simplify the term as far as possible

$$h(g(x)) = 1 - 2\left(2 - \frac{1}{4}x^3\right)^2 = 1 - 2\left(4 - x^3 + \frac{1}{16}x^6\right) =$$

$$-7 + 2x^3 - \frac{1}{8}x^6$$

Task 8. Extremal points of functions of two variables

Given is the function
$f(x,y) = 4x^2y + 2xy - 3y^2 + 5$

(a)Calculate the following partial derivatives:

$f_x = 8xy + 2y$
$f_y = 4x^2 + 2x - 6y$
$f_{xx} = 8y$
$f_{xy} = 8x + 2$
$f_{yy} = -6$

(b)Calculate all critical points (x,y) of f (i.e., all points where f_x and f_y are both 0)

$$\begin{cases} 8xy + 2y = 0 \\ 4x^2 + 2x - 6y = 0 \end{cases}$$

$$2y(4x+1) = 0 \ \rightarrow \ y = 0; x = -\frac{1}{4}$$

$$y = 0 \quad x = 0 \ \ or \ x = -\frac{1}{2}$$

$$x = -\frac{1}{4} \ y = -\frac{1}{24}$$

(c) Indicate for each critical point if it is a saddle point or a local extremal point of f , and in the latter case, if it is a maximum or a minimum

$$H(f) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 8y & 8x+2 \\ 8x+2 & -6 \end{pmatrix}$$

$\boldsymbol{y = 0; \ x = 0}$

$$D = f_{xx}(x_0, y_0) \cdot f_{yy}(x_0, y_0) - f_{xy}(x_0, y_0) \cdot f_{yx}(x_0, y_0) =$$

$$0 \cdot (-6) - 2 \cdot 2 = -4 < 0 \rightarrow saddle\ point$$

$$\boldsymbol{x = -\frac{1}{2}; y = 0}$$

$$D = f_{xx}(x_0, y_0) \cdot f_{yy}(x_0, y_0) - f_{xy}(x_0, y_0) \cdot f_{yx}(x_0, y_0) =$$

$$0 \cdot (-6) - (-2) \cdot (-2) = -4 < 0 \rightarrow saddle\ point$$

$$x = -\frac{1}{4}; y = -\frac{1}{24}$$

$$D = f_{xx}(x_0, y_0) \cdot f_{yy}(x_0, y_0) - f_{xy}(x_0, y_0) \cdot f_{yx}(x_0, y_0) =$$

$$8\left(-\frac{1}{24}\right) \cdot (-6) - \left(8\left(-\frac{1}{4}\right) + 2\right)^2 = 2 > 0$$

$$f_{xx}(x_0, y_0) = -\frac{1}{3} < 0 \rightarrow local\ maximum$$

Task 9 Integration

Calculate the values of the following integrals:

(a)
$$\int_0^2 (x^3 - x)\,dx = \frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_0^2 = \frac{1}{4}16 - \frac{1}{2}4 - 0 = 2$$

(b)

$$\int_0^{\pi} sinx\,dx = -cosx\Big|_0^{\pi} = -cos\pi - (-cos0) = 1 + 1 = 2$$

Task 10

(a)

A B C D
A B D C
A C B D
A C D B
A D B C
A D C B
B A C D
B A D C
B C A D
B C D A
B D A C
B D C A
C A B D
C A D B
C B A D
C B D A
C D A B
C D B A
D A B C
D A C B
D B A C
D B C A
D C A B
D C B A

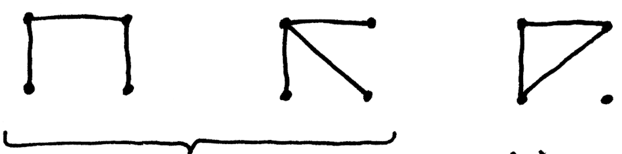
$(P(4) = 24)$

(b) Each of the n letters can occur in the first position.
The other $(n-1)$ letters can occur in $P(n-1)$ permutations on the rest of the positions.
Together, there are $n \cdot P(n-1)$ possibilities to permute all n letters.

$\Rightarrow \quad P(n) = n \cdot P(n-1).$

(From this, it follows: $P(n) = n!$.)

Task 11



2 are connected

1 is unconnected

Task 12

(a) $8 \cdot 10^6 \cdot 2 \text{ bit} \;=\; 2 \cdot 10^6 \text{ Byte} \;=\; 2 \text{ MB}$

(b) $\underbrace{2^{10}}_{1024} \cdot 2^{10} \cdot 24 \text{ bit} \;=\; 2^{20} \cdot 2^3 \cdot 3 \text{ bit} \;=\; 3 \cdot 2^{20} \text{ Byte} \approx 3 \cdot 10^6 \text{ Byte} = 3 \text{ MB}$

$\Rightarrow$ (b) needs more storage capacity.

Task 13

(a) x + Math.sqrt(1 - x*x)/2 $\longrightarrow$ $x + \frac{\sqrt{1-x^2}}{2}$

(b) possible runtime error: root of negative number.
Condition to be checked to avoid this: $1 - x^2 \geq 0$
(or, equivalently: $x^2 \leq 1$, or: $|x| \leq 1$.)

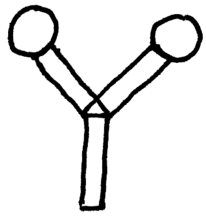
Task 14

Bud ==> Shoot [RU(45) Bud] [RU(-45) Bud] ;

after a second application of this rule we get:

Shoot [RU(45) Shoot [RU(45) Bud][RU(-45) Bud]]
[RU(-45) Shoot [RU(45) Bud][RU(-45) Bud]]

With initially vertical direction of Shoot (i.e., F), the geometrical interpretation looks like this:



Task 15 *Statistics / location and dispersion measures*

Given are heights of trees in m (n = 11 trees):

5.9 7.5 6.6 10.2 7.8 9.4 8.5 7.2 9.1 9.6 9.5

(a) Calculate the 0^{th}, 25^{th}, 50^{th}, 75^{th} and 100^{th} percentiles (named Q_0, Q_{25}, Q_{50}, Q_{75}, Q_{100}).

(b) Calculate the median and the mean.

(c) Calculate the variance, the standard deviation and the coefficient of variation.

(d) Calculate the range and interquartile range.

Solution

Ordered sequence:

5.9 6.6 7.2 7.5 7.8 8.5 9.1 9.4 9.5 9.6 10.2

$0^{\text{th}} = 5.9$

25^{th}: $t = 25;\ p = 0.25;\ n \cdot p = 11 \cdot 0.25 = 2.75;\ j = 2;\ g = 0.75;\ x_{[3]} = 7.2$

50^{th}: $t = 50;\ p = 0.50;\ n \cdot p = 11 \cdot 0.50 = 5.50;\ j = 5;\ g = 0.50;\ x_{[6]} = 8.5$

75^{th}: $t = 75;\ p = 0.75;\ n \cdot p = 11 \cdot 0.75 = 8.25;\ j = 8;\ g = 0.25;\ x_{[9]} = 9.5$

$100^{\text{th}} = 10.2$

Table:

i	x_i	x_i^2
1	5.9	34.81
2	7.5	56.25
3	6.6	43.56
4	10.2	104.04
5	7.8	60.84
6	9.4	88.36
7	8.5	72.25
8	7.2	51.84
9	9.1	82.81
10	9.6	92.16
11	9.5	90.25
$\sum_{i=1}^{n}$	91.3	777.17

Mean:

$$\bar{x} = \frac{\sum_{i=1}^{11} x_i}{11} = \frac{91.3}{11} = 8.3$$

Variance:

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}{n-1} = \frac{777.17 - \frac{(91.3)^2}{11}}{10} = 1.94$$

Standard deviation:

$$s = \sqrt{1.94} = 1.39$$

Coefficient of variation:

$$CV = \frac{s}{\bar{x}} = \frac{1.39}{8.3} \cdot 100\% = 16.8\%$$

Range:

$$V = x_{max} - x_{min} = 10.2 - 5.9 = 4.3$$

Interquartile range:

$$R_{IQ} = upper\ quatrile - lower\ quartile = 9.5 - 7.2 = 2.3$$

Task 16 *Statistics / Gaussian distribution*

Given is a field with 100.000 corn plants. The mean height of a plant is μ = 150 cm and the standard deviation of the population is σ = 25 cm. The height is approximately normally distributed. One sample containing $n = 25$ plants was taken from this population.

(a) What percentage of individual values do you expect to lie between 135 and 180 cm?

Solution

$\mu = 150;\ \sigma = 25;$

Find: $P(135 \le X \le 180)$.

$$P(135 \le X \le 180) = 1 - P(X < 135) - P(X > 180)$$

$$z_1 = \frac{(135 - 150)}{25} = -0.6;$$

$$P(X < 135) = P(Z < z_1 = -0.6) = P(Z > -z_1 = 0.6) = 0.2743$$

$$z_2 = \frac{(180 - 150)}{25} = 1.2;$$

$P(X > 180) = P(Z > z_2 = 0.2) = 0.1151$

$\Rightarrow P(135 \leq X \leq 180) = 1 - P(X < 135) - P(X > 180)$
$= 1 - 0.2743 - 0.1151 = 0.6106$

We expect 61.1% of individual values between 135 and 180 cm.

(b) What percentage of individual values do you expect to lie between 155 and 175 cm?

Solution

$\mu = 150; \sigma = 25;$

Find: $P(155 \leq X \leq 175)$.

$P(155 \leq X \leq 180) = P(X > 155) - P(X > 175)$

$z_1 = \dfrac{(155 - 150)}{25} = 0.2;$

$P(X > 155) = P(Z > z_1 = 0.2) = 0.4207$

$z_2 = \dfrac{(175 - 150)}{25} = 1.0;$

$P(X > 175) = P(Z > z_2 = 1.0) = 0.1587$

$\Rightarrow P(155 \leq X \leq 180) = P(X > 155) - P(X > 175)$

$= 0.4207 - 0.1587 = 0.262$

We expect 26.2% of individual values between 155 and 175 cm.

(c) What percentage of sample mean values do you expect to lie between 140 and 155 cm?

Solution

$\mu = 150; \sigma = 25;$ standard error of mean $\sigma_{\bar{x}} = \frac{25}{\sqrt{25}} = 5$

Find: $P(140 \leq \bar{X} \leq 155)$.

$P(140 \leq X \leq 155) = 1 - P(X < 140) - P(X > 155)$

$$z_1 = \frac{(140 - 150)}{5} = -2.0;$$

$$P(X < 140) = P(Z > -z_1) = 0.0228$$

$$z_2 = \frac{(155 - 150)}{5} = 1.0;$$

$$P(X > 155) = P(Z > z_2) = 0.1587$$

$$\Rightarrow P(140 \leq X \leq 155) = 1 - P(X < 140) - P(X > 155)$$

$$= 1 - 0.0228 - 0.1587 = 0.8185$$

We expect 81.9% of sample means between 140 and 155 cm.

(d) What percentage of sample mean values do you expect to lie between 140 and 145 cm?

Solution

$\mu = 150$; $\sigma = 25$; Standard error of mean $\sigma_{\bar{x}} = \frac{25}{\sqrt{100}} = 2.5$

Find: $P(140 \leq \bar{X} \leq 145)$.

$$P(140 \leq X \leq 145) = P(X < 145) - P(X < 140)$$

$$z_1 = \frac{(140 - 150)}{5} = -2;$$

$$P(X < 140) = P(Z > -z_1) = 0.0228$$

$$z_2 = \frac{(145 - 150)}{5} = -1.0;$$

$$P(X < 145) = P(Z > -z_2) = 0.1587$$

$$\Rightarrow P(140 \leq X \leq 145) = P(X < 145) - P(X < 140)$$

$$= 0.1587 - 0.0228 = 0.1359$$

We expect 13.6% of sample means between 140 and 145 cm.

Task 17 *Statistics / Confidence intervals, one-sided Gauss test*

A sample of size $n = 25$ was taken from a population of corn plants. The measured average height of the 25 plants was $\bar{x} = 155$ cm and the standard deviation of the population is $\sigma =$ 25 cm. The height is approximately normally distributed.

(a) Compute a 99% confidence interval for the mean height.

Solution:

99% confidence interval for the mean height:

$$lower\ limit: \bar{x} - z_{1-\alpha/2} \cdot \sigma_{\bar{x}} = 155 - 2.57 \cdot 5 = 142.15$$

$$upper\ limit: \bar{x} + z_{1-\alpha/2} \cdot \sigma_{\bar{x}} = 155 + 2.57 \cdot 5 = 167.85$$

With the probability of 99% the mean height lies between 142.15 and 167.85 cm.

(b) Formulate the null hypothesis and answer the question whether the mean of the population from which the sample was taken from is significantly higher than 145 cm ($\alpha = 0.05$).

Solution:

$H_0: \mu \leq 145$
$H_0: \mu > 145$

One-tailed test

$$z_{Test} = \frac{\bar{x} - \mu_0}{\sqrt{\sigma^2/n}}$$

$\bar{x} = 155;\ s = 25\ cm$

$$z_{Test} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{155 - 145}{25/\sqrt{25}} = 2.0$$

$\alpha = 0.05 \Rightarrow z_{1-\alpha} = 1.64$

$z_{Test} > z_{Tab} \Rightarrow H_0$ is rejected

The measured height is significantly higher, than 145 cm.

(c) Formulate the null hypothesis and answer the question whether the mean of the population from which the sample was taken from is significantly lower than 160 cm (α = 0.05).

Solution:

$$H_0 : \mu \geq 160$$
$$H_0 : \mu < 160$$

One-tailed test

$$z_{Test} = \frac{\bar{x} - \mu_0}{\sqrt{\sigma / n}}$$

$$\bar{x} = 155;\ \sigma = 25\ cm$$

$$z_{Test} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{155 - 160}{25/\sqrt{25}} = -1.0$$

$$\alpha = 0.05 \Rightarrow z_\alpha = -1.64$$

$z_{Test} > z_{Tab} \Rightarrow H_0$ is not rejected

The measured average height is not significantly lower, than 160 cm at $\alpha = 0.05$.

Task 18 *Statistics / Linear regression and correlation, coefficient of determination*

Investigated was the relationship between the height of trees in m (y) and their diameter in cm (x).
The estimated regression equation was as follows: $\hat{y} = 1.4 + 0.6\, x$,
and the estimate of the coefficient of correlation was $r = 0.91$.

(a) What height do you expect for a diameter of 10 cm?

Solution:

$$\hat{y} = 1.4 + 0.6 \cdot 10 = 7.4$$

(b) What percent of variability of y can be explained by x ?
Give your answer in words.

Solution:

$$R^2 = 0.91^2 = 0{,}83$$

About 83% of the total variance of the variable Height can be explained by the effect of the variable diameter.