Computer Science and Mathematics
Summer term 2026

Exercises 4

1. The points $A = (1; 3)$, $B = (11; 7)$ and $C = (3; 13)$ are given in the cartesian coordinate system.
   (a) Let A be the new zero (origin) and calculate the vectors $\vec{b} = \overrightarrow{AB}$ and $\vec{c} = \overrightarrow{AC}$.
   (b) Calculate the vector $\vec{d} = \vec{b} + \vec{c} = \overrightarrow{AD}$ and the absolute coordinates of the new point D.
   (c) Calculate the inner product $\vec{b} \cdot \vec{c}$ and the angle $\angle(\vec{b}, \vec{c})$.
   (d) Extend the vectors by a third dimension (with value 0) and calculate the cross product $\vec{b} \times \vec{c}$.
   (e) Calculate the area of the parallelogram spanned by $\vec{b}$ and $\vec{c}$.

2. Let p be the plane in $\mathbb{R}^3$ which goes through the points $A = (7; 1; 5)$, $B = (8; 3; 5)$ and $C = (10; 1; 1)$. Calculate a vector which is orthogonal to p.

3. Let $f\colon\ \mathbb{R}^2 \to \mathbb{R}^2$ be the linear mapping which performs a counterclockwise rotation by 45° around (0; 0). What is the matrix of f?
   (Hint: Remember that its columns are the images of the standard basis vectors under f.)

4. Let $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.
   (a) Calculate $A \cdot \vec{e}_1$, $A \cdot \vec{e}_2$, $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, and $A \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.
   (b) Give a geometrical description of the linear mapping $\vec{x} \mapsto A \cdot \vec{x}$ which is associated to A.

5. Determine the results:
   (a) $3 \cdot \begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & 2 \\ 2 & 5 & 0 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 0 \\ 7 & 0 & 0 \\ 1 & 3 & 1 \end{pmatrix}^T$
   (b) $\begin{pmatrix} 2 & 4 & 0 \\ -3 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$
   (c) $\begin{pmatrix} 1 & 1 \\ 3 & 4 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -7 \end{pmatrix}$
   (d) $rank \begin{pmatrix} -2 & 1 & 6 \\ 0 & 5 & 5 \\ 1 & -\frac{1}{2} & -3 \end{pmatrix}$