Computer Science and Mathematics
Summer term 2026

Exercises 3

1. (a) Calculate the decimal value of the binary number 1001111.
   (b) Calculate the hexadecimal representation of the decimal number 999.
   (c) What is the binary expansion of the value 1/3 ?
   (Hint: You can do "written division" analogously to the decimal case, but with doubling the remainder in every step instead of multiplying by 10.)

2. Let $\vec{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.
   (a) Draw $\vec{a}$ and $\vec{b}$ in a coordinate system.
   (b) Determine $2 \cdot \vec{a} + \vec{b}$ by calculation and graphically.
   (c) Let $\vec{c} = \begin{pmatrix} 1 \\ -11 \end{pmatrix}$. Find a representation of $\vec{c}$ as a linear combination of $\vec{a}$ and $\vec{b}$.

3. Let $\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. What geometrical objects are described by the following sets?
   (a) $\{ \vec{b} + t \cdot \vec{a} \mid t \in \mathbb{R} \wedge t \geq 0 \}$
   (b) $\{ \vec{x} \in \mathbb{R}^2 \mid \vec{a} \cdot \vec{x} = 0 \}$
   (c) $\{ \vec{x} \in \mathbb{R}^2 \mid \left\| \vec{x} - \vec{b} \right\| = 0.5 \}$

4. (a) Are the vectors $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -3 \\ -2 \\ 5 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$ linearly independent?
   (b) What is the maximal number of vectors which can be linearly independent in $\mathbb{R}^4$?