

## Exercises 2

1. Which of the following functions  $f: \mathbb{R} \rightarrow \mathbb{R}$  are bijective?

$$f(x) = 1$$

$$f(x) = 2x$$

$$f(x) = x^2$$

$$f(x) = 2^x$$

$$f(x) = x^3$$

2. Determine a formula for the inverse function  $f^{-1}$  for  $f(x) = 1/(2x+1)$  (with  $x > -1/2$ ).

3. The following sets are sets of number pairs, i.e., subsets of the cartesian product

$$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 :$$

$$A := \{ (x, y) \in \mathbb{R}^2 \mid y = \frac{2}{3}x - 2 \}$$

$$B := \{ (x, y) \in \mathbb{R}^2 \mid y = -|0.5x| + 1 \}$$

$$C := \{ (x, y) \in \mathbb{R}^2 \mid x \geq 0 \wedge -x+3 \geq y \geq 0 \}$$

- (a) Visualize each of the sets  $A, B, C$  in the cartesian coordinate system. (Make a separate graphical image for each set.)  
(b) All these sets are relations. Which of them are even functions?

4. Let  $P$  be the set of all participants of a party and  $D$  the set of all drinks which are offered there. Each participant  $p_i \in P$  shall get a drink  $d_i \in D$  according to his/her preference. In this way, a mapping  $f: P \rightarrow D$  shall be defined.

What do the following possible properties of  $f$  mean in this context?

- (a) surjectivity                      (c) bijectivity  
(b) injectivity                        (d) Can a participant of the party get two different drinks?

5. Let  $f: D \rightarrow R$  be the function described by  $f(x) = 2x^3 - 1$ .

The domain  $D$  is defined as  $D := \{ x \in \mathbb{R} \mid -3 \leq x < 2 \}$ . Determine

- (a) the range  $R := f(D)$ ,  
(b) a formula for the inverse function  $f^{-1}$ .