Exercises 8

Task 1

Given is the following function of two variables:

$$f(x, y) = 18x^2 - 36xy - 6y^3 - 36x + 20y.$$

(a) Calculate the first and second partial derivatives of f (i.e., the functions f_x, f_y, f_{xx}, f_{xy}, f_{yy}).

(b) Determine all critical points of f and make decisions about possible extrema (if maximum or minimum).

Task 2

Determine the following indefinite integrals (if necessary, use a table of integrals):

(a) $\int (5x^4 - 3 + \frac{2}{\sqrt{x}})\, dx$

(b) $\int \frac{4x^3 - \frac{1}{x}}{x^4 - \ln x}\, dx$

(c) $\int \frac{7}{\sqrt{4 - t^2}}\, dt$

(d) $\int 2^{(x-1)}\, dx$

(e) $\int \frac{2}{\cos^2(2x)}\, dx$

Task 3

Calculate the following definite integrals (trigonometric functions should be evaluated with angle specifications in radians):

(a) $\int\limits_0^1 (e^x + \frac{3}{2}x^2)\, dx$

(b) $\int\limits_{-\pi/4}^{\pi/4} \sin x\, dx$

(c) $\int\limits_{41}^{42} x\, dx$

(d) $\int\limits_{-1}^{1} \frac{1}{9 - x^2}\, dx$

Task 4

Calculate the area which is enclosed from above by the graph of the function $f(x) = 4x - x^2 - 3$, below by the graph of the function $g(x) = x - 3$, from the left by $x = 1$ and from the right by $x = 3$.