Exercises 6

1.

Determine the first derivative of the following functions *f*. Apply, where necessary, product rule, quotient rule and chain rule. In these cases, please determine first their components in the given situation. Simplify the results as far as possible.

(a) $f(x) = x^5 - 3x^4 + \sqrt[3]{x}$

(b) $f(x) = \ln(\ln x)$

(c) $f(x) = (x^2 + 1) \cdot \sin x$

(d) $f(x) = \dfrac{2x+1}{x^2+3}$

(e) $f(x) = e^{\frac{1}{x}}$

2.

For a growth function f there was the assumption that up to an age of $t = 10$ years, there is a linear increase according to $k \cdot t$ (with a constant $k > 0$), while afterwards a behavior according to $4\sqrt{t+6} - 11$ is taken:

$$f(t) = \begin{cases} kt & \text{for } 0 \le t \le 10, \\ 4 \cdot \sqrt{t+6} - 11 & \text{for } t > 10. \end{cases}$$

(a) How must the number k be chosen in order to let the function f be contiuous at $t = 10$?

(b) Calculate (for *this k*) the left-sided and the right-sided derivative of f at $t = 10$. Is f differentiable at this point?

3.

Given is the polynomial function $f(x) = x^3 - 7.5\,x^2 + 18x - 1$.

(a) Compute all values x where the first derivative of f has the value 0 ("critical points" of f).

(b) Draw a graph of the function f on the interval [0; 4].
What happens with f at the critical points from (a)?