Computer Science and Mathematics
Summer term 2024

Exercises 3

1. Let $\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. What geometrical objects are described by the following sets?

   (a) $\{ \ \vec{b} + t \cdot \vec{a} \ \mid t \in \mathbb{R} \ \wedge \ t \geq 0 \ \}$
   (b) $\{ \ \vec{x} \in \mathbb{R}^2 \mid \ \vec{a} \cdot \vec{x} = 0 \ \}$
   (c) $\{ \ \vec{x} \in \mathbb{R}^2 \mid \ \left\| \vec{x} - \vec{b} \right\| = 0.5 \ \}$

2. (a) Are the vectors $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -3 \\ -2 \\ 5 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$ linearly independent?

   (b) What is the maximal number of vectors which can be linearly independent in $\mathbb{R}^4$?

3. The points $A = (1; 3)$, $B = (11; 7)$ and $C = (3; 13)$ are given in the cartesian coordinate system.

   (a) Let A be the new zero (origin) and calculate the vectors $\vec{b} = \overrightarrow{AB}$ and $\vec{c} = \overrightarrow{AC}$.
   (b) Calculate the vector $\vec{d} = \vec{b} + \vec{c} = \overrightarrow{AD}$ and the absolute coordinates of the new point D.
   (c) Calculate the inner product $\vec{b} \cdot \vec{c}$ and the angle $\angle(\vec{b}, \vec{c})$.
   (d) Extend the vectors by a third dimension (with value 0) and calculate the cross product $\vec{b} \times \vec{c}$.
   (e) Calculate the area of the parallelogram spanned by $\vec{b}$ and $\vec{c}$.

4. Let p be the plane in $\mathbb{R}^3$ which goes through the points $A = (7; 1; 5)$, $B = (8; 3; 5)$ and $C = (10; 1; 1)$. Calculate a vector which is orthogonal to p.

5. Let $f\colon \ \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear mapping which performs a counterclockwise rotation by 45° around (0; 0). What is the matrix of f?
   (Hint: Remember that its columns are the images of the standard basis vectors under f.)