

## Exercises 1

1. Show with Venn diagrams:

(a)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b)  $(A \cap B)^c = A^c \cup B^c$

2. (a) How many subsets with exactly 3 elements does a set with 5 elements have?

(b) Let  $B(n, k)$  = number of  $k$ -element subsets of a set with  $n$  elements.

Show that  $B(n, k) = B(n-1, k) + B(n-1, k-1)$  if  $n \geq 1$  and  $0 < k \leq n$ .

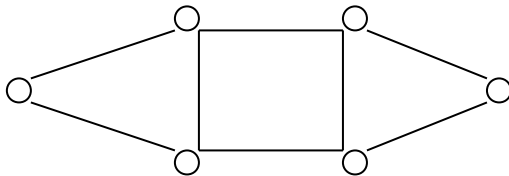
(c) From (b), deduce the list of numbers  $B(n, k)$  for  $n = 0, 1, \dots, 6$  and  $k = 0, 1, \dots, n$ .  
In which other mathematical context do they appear?

3. Find a formula for  $|A \cup B \cup C|$ .

4. What is the number of words of length  $n$  over an alphabet with  $k$  elements?

List them systematically for the case  $k = 2$ ,  $n = 4$ .

5. How many circles (with no edges appearing more than one time in it) are contained in this graph?



6. List all functions of the set  $\{a; b; c\}$  into itself.  
Which of them are bijective?

7. Simplify as far as possible:

(a) 
$$\frac{1 - \frac{x-1}{x+1}}{x - \frac{1}{x}}$$

(b)  $(a + 2b)^2 - (b - 2a)^2 + (a - b)(a + b)$

(c)  $f(g(x))$  for  $f(z) = z^3 + z + 1$  and  $g(x) = x - 1$ .