Exercises 12

Task 1

Calculate the following limits:

(a) $\lim\limits_{x\to 2} \dfrac{x^3 - 4x^2 + x + 6}{x^3 - 6x^2 + 3x + 10}$

(b) $\lim\limits_{x\to 0} \dfrac{1 - e^x}{3\sin x}$

Task 2

The following function is given: $f(x) = x^3 - 3x^2 - x + 3$.

(a) Make a table of the values of $f(x)$ for $x = -2; -1; 0; 1; 2; 3; 4$.

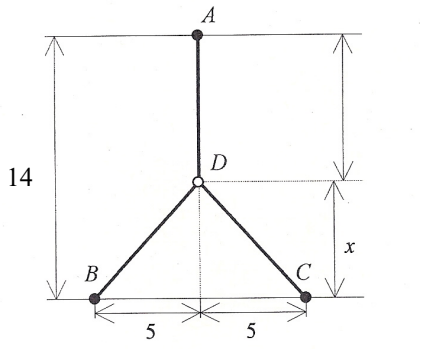
(b) Determine all intersections of the graph of f with the x axis and with the y axis.

(c) Determine the first and the second derivative $f'(x)$ and $f''(x)$.

(d) Determine the x values where the function f takes its local extrema, and for each the type of extremum (min./max.)

Task 3

In a forest, the points A, B and C shall be connected by paths (see the figure, paths indicated by bold lines, distances given in km).



In which distance x from the base line BC has the branching point D to be chosen if the costs of the path network shall be minimal? Here, the costs are assumed to be proportional to the total length $AD + DB + DC$ of the paths.

Hint: The lengths DB and DC are identical and can be determined with Pythagoras‘ theorem. Follow the approach $f(x) = $ *total length* and calculate the (positive) value where the first derivative becomes zero. The second derivative needs not to be checked since from the situation it is clear that only a minimum, not a maximum can occur here.

Task 4

Given is the following function of two variables:

$$f(x, y) = x^2 + 4y^2 - xy - 10y + 3 .$$

Calculate the first and second partial derivatives of f (i.e., the functions f_x , f_y , f_{xx} , f_{xy} , f_{yy}).