Exercises 11

1.

Determine the first derivative of the following functions f. Apply, where necessary, product rule, quotient rule and chain rule. In these cases, please determine first their components in the given situation. Simplify the results as far as possible.

(a) $f(x) = x^5 - 3x^4 + \sqrt[3]{x}$

(b) $f(x) = \ln(\ln x)$

(c) $f(x) = (x^2 + 1) \cdot \sin x$

(d) $f(x) = \dfrac{2x+1}{x^2+3}$

(e) $f(x) = e^{\frac{1}{x}}$

2.

For a growth function f there was the assumption that up to an age of $t = 10$ years, there is a linear increase according to $k \cdot t$ (with a constant $k > 0$), while afterwards a behavior according to $4\sqrt{t+6} - 11$ is taken:

$$f(t) = \begin{cases} kt & \text{for } 0 \le t \le 10, \\ 4 \cdot \sqrt{t+6} - 11 & \text{for } t > 10. \end{cases}$$

(a) How must the number k be chosen in order to let the function f be contiuous at $t = 10$?

(b) Calculate (for *this* k) the left-sided and the right-sided derivative of f at $t = 10$. Is f differentiable at this point?

3.

Given is the polynomial function $f(x) = x^3 - 7.5 \, x^2 + 18x - 1$.

(a) Compute all values x where the first derivative of f has the value 0 ("critical points" of f).

(b) Draw a graph of the function f on the interval [0; 4].
    What happens with f at the critical points from (a)?