Exercises 4

1. Let $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

   (a) Calculate $A \cdot \vec{e}_1$, $A \cdot \vec{e}_2$, $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, and $A \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

   (b) Give a geometrical description of the linear mapping $\vec{x} \mapsto A \cdot \vec{x}$ which is associated to A.

2. Determine the results:

   (a) $3 \cdot \begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & 2 \\ 2 & 5 & 0 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 0 \\ 7 & 0 & 0 \\ 1 & 3 & 1 \end{pmatrix}^{\mathrm{T}}$

   (b) $\begin{pmatrix} 2 & 4 & 0 \\ -3 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & 1 \\ 3 & 4 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -7 \end{pmatrix}$

   (d) $rank \begin{pmatrix} -2 & 1 & 6 \\ 0 & 5 & 5 \\ 1 & -\frac{1}{2} & -3 \end{pmatrix}$

3. Evaluate the following determinants:

   (a) $\begin{vmatrix} a+b & 2b \\ 2a & a+b \end{vmatrix}$

   (b) $\begin{vmatrix} 2 & 0 & 8 & 1 \\ 5 & 3 & 1 & 6 \\ 7 & 0 & 2 & 0 \\ 1 & 0 & 0 & -1 \end{vmatrix}$

4. For which of the following pairs of matrices is their product defined?
   In these cases, calculate the resulting matrix.

(a) $\begin{pmatrix} 4 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 \\ 3 & x \end{pmatrix}$

(b) $\begin{pmatrix} 5 & 3 \\ 1 & 0 \\ 8 & 7 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 0 & -6 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 10 \\ 0 & 2 & 0 \\ 1 & 5 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 30 \\ 20 \\ 10 \end{pmatrix}$

(e) $\begin{pmatrix} 50 \\ 60 \end{pmatrix} \cdot \begin{pmatrix} -2 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$