Computer Science and Mathematics Summer term 2023

Exercises 3

- 1. Let $\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. What geometrical objects are described by the following sets?
 - (a) $\{\vec{b} + t \cdot \vec{a} \mid t \in \mathbb{R} \land t \ge 0\}$
 - (b) { $\vec{x} \in \mathbb{R}^2 | \vec{a} \cdot \vec{x} = 0$ }
 - (c) { $\vec{x} \in \mathbb{R}^2 | ||\vec{x} \vec{b}|| = 0.5 }$
- 2. (a) Are the vectors $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -3 \\ -2 \\ 5 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$ linearly independent?
 - (b) What is the maximal number of vectors which can be linearly independent in \mathbb{R}^4 ?
- 3. The points A = (1; 3), B = (11; 7) and C = (3; 13) are given in the cartesian coordinate system.
 - (a) Let A be the new zero (origin) and calculate the vectors $\vec{b} = \overrightarrow{AB}$ and $\vec{c} = \overrightarrow{AC}$.
 - (b) Calculate the vector $\vec{d} = \vec{b} + \vec{c} = \overrightarrow{AD}$ and the absolute coordinates of the new point D.
 - (c) Calculate the inner product $\vec{b} \cdot \vec{c}$ and the angle $\angle(\vec{b}, \vec{c})$.
 - (d) Extend the vectors by a third dimension (with value 0) and calculate the cross product $\vec{b} \times \vec{c}$.
 - (e) Calculate the area of the parallelogram spanned by $\, \vec{b} \,$ and $\, \vec{c} \,$.
- 4. Let p be the plane in \mathbb{R}^3 which goes through the points A = (7; 1; 5), B = (8; 3; 5) and C = (10; 1; 1). Calculate a vector which is orthogonal to p.
- 5. Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear mapping which performs a counterclockwise rotation by 45° around (0; 0). What is the matrix of f?

(Hint: Remember that its columns are the images of the standard basis vectors under f.)