Computer Science and Mathematics Summer term 2023

Exercises 2

- 1. Which of the following functions $f: \mathbb{R} \to \mathbb{R}$ are bijective?
 - f(x) = 1
 - f(x) = 2x
 - $f(x) = x^2$
 - $f(x) = 2^x$
 - $f(x) = x^3$
- 2. Determine a formula for the inverse function f^{-1} for f(x) = 1/(2x+1) (with x > -1/2).
- 3. The following sets are sets of number pairs, i.e., subsets of the cartesian product $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$:
 - $A := \{ (x, y) \in \mathbb{R}^2 \mid y = \frac{2}{3}x 2 \}$
 - $B := \{ (x, y) \in \mathbb{R}^2 \mid y = -|0.5x| + 1 \}$
 - $C := \{ (x, y) \in \mathbb{R}^2 \mid x \ge 0 \land -x+3 \ge y \ge 0 \}$
 - (a) Visualize each of the sets A, B, C in the cartesian coordinate system. (Make a separate graphical image for each set.)
 - (b) All these sets are relations. Which of them are even functions?
- 4. Let P be the set of all participants of a party and D the set of all drinks which are offered there. Each participant $p_i \in P$ shall get a drink $d_i \in D$ according to his/her preference. In this way, a mapping $f: P \to D$ shall be defined.

What do the following possible properties of f mean in this context?

- (a) surjectivity
- (c) bijectivity
- (b) injectivity
- (d) Can a participant of the party get two different drinks?
- 5. Let $f: D \to R$ be the function described by $f(x) = 2x^3 1$.

The domain *D* is defined as $D := \{ x \in \mathbb{R} \mid -3 \le x < 2 \}$. Determine

- (a) the range R := f(D),
- (b) a formula for the inverse function f^{-1} .
- 6. (a) Calculate the decimal value of the binary number 1001111.
 - (b) Calculate the hexadecimal representation of the decimal number 999.
 - (c) What is the binary expansion of the value 1/3?

(Hint: You can do "written division" analogously to the decimal case, but with doubling the remainder in every step instead of multiplying by 10.)

- 7. Let $\vec{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.
 - (a) Draw \vec{a} and \vec{b} in a coordinate system.
 - (b) Determine $2 \cdot \vec{a} + \vec{b}$ by calculation and graphically.
 - (c) Let $\vec{c} = \begin{pmatrix} 1 \\ -11 \end{pmatrix}$. Find a representation of \vec{c} as a linear combination of \vec{a} and \vec{b} .