

Exercises 1

1. Show with Venn diagrams:

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $(A \cap B)^c = A^c \cup B^c$

2. (a) How many subsets with exactly 3 elements does a set with 5 elements have?

(b) Let $B(n, k)$ = number of k -element subsets of a set with n elements.

Show that $B(n, k) = B(n-1, k) + B(n-1, k-1)$ if $n \geq 1$ and $0 < k \leq n$.

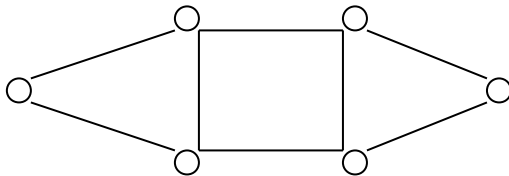
(c) From (b), deduce the list of numbers $B(n, k)$ for $n = 0, 1, \dots, 6$ and $k = 0, 1, \dots, n$.
In which other mathematical context do they appear?

3. Find a formula for $|A \cup B \cup C|$.

4. What is the number of words of length n over an alphabet with k elements?

List them systematically for the case $k = 2$, $n = 4$.

5. How many circles (with no edges appearing more than one time in it) are contained in this graph?



6. List all functions of the set $\{a; b; c\}$ into itself.
Which of them are bijective?

7. Simplify as far as possible:

(a)
$$\frac{1 - \frac{x-1}{x+1}}{x - \frac{1}{x}}$$

(b) $(a + 2b)^2 - (b - 2a)^2 + (a - b)(a + b)$

(c) $f(g(x))$ for $f(z) = z^3 + z + 1$ and $g(x) = x - 1$.