Computer Science and Mathematics
Summer term 2019

Exercises 5

1. A tree stand at time *t* consists of 20 ha of young trees (age class 1), 40 ha of medium-age trees (class 2) and 40 ha of old trees (class 3).
After 1 time step, the following events have occurred:

- 25 % of the young trees have been killed by the Dark Monster Beetle,
- 10 % of the medium-age trees have been lethally damaged by the Wild Crazy Deer,
- all old trees have been harvested,
- all damaged trees have been removed,
- all open spaces have been reforested with young trees,
- all other trees have grown older into the next age class.

Give the transition matrix of this age-class dynamics and calculate the age-class vector of the stand at time *t*+1.

2. Do the following matrices have an inverse? If so, calculate it.

(a) $A = \begin{pmatrix} 6 & -10 \\ -9 & 15 \end{pmatrix}$

(b) $B = \begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix}$

3. Decide with Frobenius' theorem if the following system of linear equations has a unique solution:

$$\begin{aligned} x - 5y &= 4z \\ x - 2y + 2z &= 1 \\ 3y - z &= 15 \end{aligned}$$

4. Determine the eigenvalues of the matrix $A = \begin{pmatrix} 5 & 7 \\ 3 & 1 \end{pmatrix}$.