Computer Science and Mathematics
Summer term 2019

Exercises 1

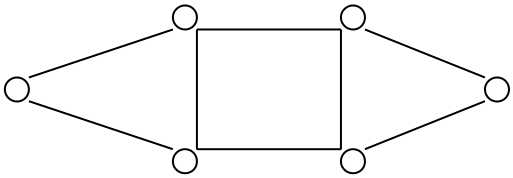
1. Show with Venn diagrams:
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (b) $(A \cap B)^C = A^C \cup B^C$

2. (a) How many subsets with exactly 3 elements does a set with 5 elements have?
   (b) Let $B(n, k)$ = number of k-element subsets of a set with n elements.
   Show that $B(n, k) = B(n–1, k) + B(n–1, k–1)$ if $n \geq 1$ and $0 < k \leq n$.
   (c) From (b), deduce the list of numbers $B(n, k)$ for $n = 0, 1, ..., 6$ and $k = 0, 1, ...$ n.
   In which other mathematical context do they appear?

3. Find a formula for $|A \cup B \cup C|$.

4. What is the number of words of length n over an alphabet with k elements?
   List them systematically for the case $k = 2$, $n = 4$.

5. How many circles (with no edges appearing more than one time in it) are contained in this graph?



6. List all functions of the set $\{ a; b; c \}$ into itself.
   Which of them are bijective?

7. (just for training) Simplify as far as possible:
   (a) $\dfrac{1-\dfrac{x-1}{x+1}}{x-\dfrac{1}{x}}$

   (b) $(a + 2b)^2 – (b – 2a)^2 + (a – b)(a + b)$

   (c) $f(g(x))$ for $f(z) = z^3 + z + 1$ and $g(x) = x – 1$.