Computer Science and Mathematics
Summer term 2018

Exercise Sheet 4

0. Prove that the ***column rank*** always equals to the ***row rank*** for any matrix.

1. Find inverses of the following matrices:

(a) $A = \begin{pmatrix} 6 & -10 \\ -9 & 15 \end{pmatrix}$

(b) $B = \begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix}$

2. Solve the following system of linear equations using **Gaussian Elimination**:

$$\begin{aligned} x + 2y + 3z &= -7 \\ 2x - 3y - 5z &= 9 \\ z - 6x - 8y + 22 &= 0 \end{aligned}$$

3. The points $A = (1; 3)$, $B = (11; 7)$ and $C = (3; 13)$ are given in the cartesian coordinate system.
   (a) Let A be the new zero (origin) and calculate the vectors $\vec{b} = \overrightarrow{AB}$ and $\vec{c} = \overrightarrow{AC}$.
   (b) Calculate the vector $\vec{d} = \vec{b} + \vec{c} = \overrightarrow{AD}$ and the absolute coordinates of the new point D.
   (c) Calculate the inner product $\vec{b} \cdot \vec{c}$ and the angle $\angle(\vec{b}, \vec{c})$.
   (d) Extend the vectors by a third dimension (with value 0) and calculate the cross product $\vec{b} \times \vec{c}$.
   (e) Calculate the area of the parallelogram spanned by $\vec{b}$ and $\vec{c}$.

4. Determine the eigenvalues of the matrix $A = \begin{pmatrix} 5 & 7 \\ 3 & 1 \end{pmatrix}$.