Computer Science and Mathematics
Summer term 2018

Exercise Sheet 2

0. Show that if you have <u>any</u> graph <u>with 3 or less cycles</u>, then it is necessarily **planar**.

1. Which of the following functions $f\colon \mathbb{R} \rightarrow \mathbb{R}$ are bijective?

$f(x) = 1$
$f(x) = 2x$
$f(x) = x^2$
$f(x) = 2^x$
$f(x) = x^3$
$f(x) = 1/ln(x)$
Bonus case ☺: $f(x)^2 = x$

2. Determine a formula for the **inverse function** f^{-1} if $f(x) = 1/(2x+1)$ (with $x > -1/2$).

3. The following sets are sets of number pairs, i.e., subsets of the Cartesian product $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$:

$A := \{ (x, y) \in \mathbb{R}^2 \mid y = \frac{2}{3}x - 2 \}$
$B := \{ (x, y) \in \mathbb{R}^2 \mid y = -|0.5x| + 1 \}$
$C := \{ (x, y) \in \mathbb{R}^2 \mid x \geq 0 \wedge -x+3 \geq y \geq 0 \}$

(a) Visualize each of the sets A, B, C in the Cartesian coordinate system
(make a separate graphical image for each set).
(b) All these sets are relations. Which of them are **even functions**?

4. Let P be the set of all participants of a party and D the set of all drinks which are offered there. Each participant $p_i \in P$ gets a drink $d_i \in D$ according to his/her preference.
In this way, a mapping $f\colon P \rightarrow D$ could be defined.
What do the following possible properties of f mean in this context?
(a) **surjectivity**
(b) **injectivity**
(c) **bijectivity**
(d) Can a participant of the party get two different drinks?

5. Let $f : D \rightarrow R$ be the function described by $f(x) = 2x^3 - 1$.
The **domain** D is defined as $D := \{ x \in \mathbb{R} \mid -3 \leq x < 2 \}$.
Determine
(a) the **range** $R := f(D)$,
(b) a formula for the inverse function f^{-1} .

6. Go through the following steps with all the items (a)-(c) listed below:
- Figure out if the given relation is **reflexive, transitive, symmetric, antisymmetric**.
- Express the given relation as a **bipartite graph**.
- Determine if the given relation is a function.
- Compute the domain and the range.

(a) $\{(-2,1), (0,3), (5,4), (1,-2), (4,5), (3,0)\}$
(b) $\{(0,0), (0,2), (2,2)\}$
(c) $\{(-1,5), (0,3), (2,3), (3,-1)\}$