Computer Science and Mathematics
Summer term 2018

Exercise Sheet 1

0. Check out if the following holds for any sets A, B, C, D:
   (a) $(A \cap B) \times (C \cap D) \neq (A \times C) \cap (B \times D)$
   (b) $(B \backslash A) \cup C = (B \cup C) \backslash (A \backslash C)$

1. Compute the following set explicitly *(i.e. make out its list of elements)*:
   $\mathcal{P}\left(\mathcal{P}(\{\emptyset, \{\emptyset\}\})\right)$,
   whereby $\mathcal{P}(A)$ goes for the "power set" of *A (i.e. the set of all its subsets)* and $\emptyset$ stands for the empty set *(i.e. the set with no elements whatsoever)*.

Tips:
- ✓ it's better to rewrite every instance of empty set as { } instead of using the conventional notation $\emptyset$ - by using { }, you can easily avoid some common mistakes *(like forgetting some pairs of curly brackets or even the general fact that every subset is a set on its own right)*. The nice thing with the usage of { } is that every single element of our power set will start and end with curly brackets.
- ✓ Make sure you don't forget that every set has the empty set and itself as subsets.

2. (a) How many subsets with exactly 3 elements does a set with 5 elements have?
   (b) Let $B(n, k)$ = number of k-element subsets of a set with n elements.
   Show that $B(n, k) = B(n–1, k) + B(n–1, k–1)$ if $n \geq 1$ and $0 < k \leq n$.
   (c) From (b), deduce the list of numbers $B(n, k)$ for $n = 0, 1, ..., 6$ and $k = 0, 1, ... n$.
   In which other mathematical context do they appear?

3. Find a formula for $|A \cup B \cup C|$.
A, B, C are any sets, $|S|$ counts all the *(distinct!)* elements of a set S.

4. Start drawing an empty graph with 6 vertices and without any edges between them. Then try to add as many edges as possible such that our arising graph remains ***triangle free*** *(i.e. with no small cycles of length 3)*.

5. We have the ***complete graph*** on $n > 3$ vertices, and we try to remove as few edges as possible such that our graph becomes triangle free again. Prove that we need to remove at least $1/3$ of all its initial amount of edges.

6. List all functions of the set $\{a, b, c\}$ into itself.
   Which of them are bijective?

That's it for this week - *Viel Spaß*