Part II: Computer Science Essentials

1. Introduction to computer science

Fundamental notions, systematical overview

What is "Computer Science" / "Informatics" ?

"Computer Science" – science about a tool?

better names would be: "science of computing" or "data processing science" (focuses on activity instead of tool)

"Informatics": continental-European for "computer science"

- French: "Informatique" (since 1960s)

- German: "Informatik"

Definition: "Science of the systematical processing of information, especially the automatic processing by use of digital computers".

Latin "informare":

to give structure to something; to educate; to picture

Information:

- independent fundamental entity of the world besides matter and energy
- depends on previous knowledge of the receiver of the information
- various approaches to quantify it
- we can consider information simply as "interpreted data".

Data: represented information

(e.g. text in a book, magnetic patterns on a harddisk, ...)

But:

Hermeneutics – "the art of interpretation" – is *not* part of informatics, despite its name. Social and cultural aspects of information are largely ignored.

"Computer": comes from "to compute"

= "to calculate".

"Algorithm":

The word comes from the Persian textbook writer *Abu Ja'far Mohammed ibn Mûsâ <u>al-Khowârizmî</u> (= "father of Ja'far Mohammed, son of Moses, coming from Khowârizm" – a town in Usbekistan, today called <i>Khiva*.)

Al-Khowârizmî lived in Bhagdad, in the "House of Wisdom"

wrote book about calculation:

"Kitab al jabr w'al-muqabala" (= "rules of

reconstitution and reduction")

- here the word "algebra" comes from!

Modern meaning of "algorithm":

Finite set of rules which specify a sequence of operations in order to solve a certain problem, with the following properties:

1. **Termination**: An algorithm must come to an end after a finite number of steps.

2. **Definitness**: Each step must be defined precisely.

3. **Input**: An algorithm *can* need input values (e.g. numbers).

4. **Output**: An algorithm *must* give one or more output values.

5. **Feasibility**: An algorithm must be feasible; e.g., no intermediate step must depend on the solution of some still unsolved mathematical problem.

(after Knuth 1973)

"*Programme*" (in American English: "program"):

Version of an algorithm which can be read, interpreted and carried out by a computer.

Programming languages were designed to write precise programmes (more precise than possible in our natural language!) suitable for computers.

Some notes concerning the history of *programming:*

Early phases of computer history: *Hardware* (= the machines) was in focus (reason for the name "computer science") Later: *Software* (= programmes) increasingly important, increasingly expensive in comparison to hardware.

First "programmer": Was a woman (**Lady Ada Lovelace**, daughter of the poet Lord Byron): Developed programs for Babbage's (nonfunctional) "analytical engine"

An early concept for a programming notation was the "Plankalkül" (Zuse 1944), but it was not used in practice.

Programming these machines: Started with today so-called "machine languages" and "assembler languages" (both machinespecific).

- Later: so-called "high-level languages"
- more abstraction
- better readability for humans
- trying to integrate traditional mathematical notations
- platform-independent (not specific to certain machine)

FORTRAN (1954), COBOL (1958), LISP (1960), Pascal (1971), C (1971), C++ (extension of C, 1992), Java (1995), XL (2008) ...

(later more about programming)

Subject areas of computer science

1960s/1970s: Development of specialized university curricula

Basis: Mathematics, electrical engineering; no interest in social or cultural conditions and consequences, or more specifically: in consequences for life at working place and leisure

Classical branches (from first recommendations for curricula in the 1960s): (a) **theoretical** informatics, (b) **technical** informatics, (c) **practical** informatics, (d) **applied** informatics

Theoretical informatics: mathematical basis: not general "theory" (which would include disciplines from the humanities and social sciences relevant to informatics), but specialized "mathematical base". Example questions:

Which problems can in principle be solved by a machine?

How can **syntax and semantics** of programming languages be described?

Which kinds of logic can be used for automatic problem solving?

How do we measure **how complicated problems are**, for example with respect to time or memory requirements?

Which kinds of problems can be solved with which abstract models of computation?

How can be the **correctness** of a program be **proved** with mathematical exactness? Technical informatics: focused on hardware. Example questions:

How can computational objects and operations be represented with **physical means**?

Which are the **basic parts** from which a computer should be built?

Which are the appropriate architectural decisions for a computer?

How can a processor be organized in order to execute a special kind of program especially quickly?

How is information stored for quick access with small cost?

Which are the technical conditions for building **networks** from separate computers?

How do we build computers which survive some defects?

Practical informatics: **non application specific programming**. Example questions:

Which are the standard problems occurring in many application areas, and how can they be solved?

Which **data structures** allow efficient solving of problems, and which algorithms are best used on these data structures?

What types of **programming languages** are best suited to different types of problems?

How must **service programs** be organized which provide the user with an easier to use view of the machine than the bare hardware would do?

How are high-level programs **translated** into a form which can be executed by the underlying hardware?

How does one design user interfaces for end users?

How does one organize the **development process** of large software systems? ("Software engineering")

Applied informatics: programming for **specific application fields**. Example questions:

How are **graphical objects** represented in the computer, and how can the be visualized?

Which **numerical methods** exist to model states and processes happening in natural environments?

How should **data base systems** be structured to support the work processes in a company?

Which techniques exist to simulate the working of the human mind with computers?

What consequences has the use of computers for the **quality of life**, both in general and at the working place in particular?

Informatics in the social context:

What **ethical questions** arise from the use of computers, and how can they be answered?

(data security, privacy questions, computer viruses, hackers, violence-promoting games, software piracy, ownership of software and ideas, the open-source idea, use of information technology for warfare, for crime, for sexual exploitation, for terrorism...)

How does the use of computers influence our **way of thinking** (about the world, about humans, about the mind, about personal relationships of people...)?

How can computers, the Web and the "Web 2.0" (Facebook, Twitter, Wikipedia etc.) be used to improve education / autonomy of people / human rights / political participation...?

2. Representation and measurement of information

In digital computers and media, all data are represented by combinations of only 2 elementary states: 0 and 1 (can be "charged" / "not charged", "on" / "off", "magnetized" / "not magnetized", "open" / "closed", "high current" / "low current", "plus" / "minus" etc.)

The smallest amount of information is thus the *bit* (binary digit). It expresses which of two alternatives is the case. The alternatives are often written 0 and 1, or (sometimes) 0 and L.

n bits: represent one out of 2^n alternatives.

Codes

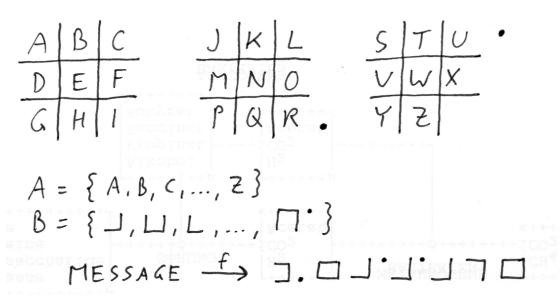
To represent information in a computer, we must *encode* all with the two symbols 0 and 1 !

What is a code?

Code (1): A mapping $f: A \rightarrow B$ from a set A of elements to be stored or transferred to a set B used for storage or transfer.

Code (2): The set *B* from definition (1).

Example:



digital (discrete) and analogue (continuous) codes

Analogue computers (representation of quantities with continuously changing quantities): have vanished

Example: Vinyl records (analogue) vs. compact disks (discrete)

Benefit of discrete data representations: avoiding noise

For digital computers, we need *binary* codes: *B* is a set of combinations of 0 and 1.

Examples:

For the primary **compass direction**: two bits necessary, and some convention which bit-pair represents which direction. Example code:

 $\{N, E, S, W\} \rightarrow \{\mathsf{0}, \mathsf{1}\}^2, N \mapsto \mathsf{00}, E \mapsto \mathsf{01}, S \mapsto \mathsf{10}, W \mapsto \mathsf{11}$

For Boolean values 'True' and 'False':

$$\{T,F\} \rightarrow \{0,1\}, T \mapsto 1, F \mapsto 0$$

For **numbers** 0 to 9: Binary Coded Decimal (BCD, non-total code, i.e. some combinations are unused)

```
\begin{array}{c} \{0,1,\ldots,9\} \to \{0,1\}^4 \\ 0 \mapsto 0000, 1 \mapsto 0001, 2 \mapsto 0010, 3 \mapsto 0011, \\ 4 \mapsto 0100, 5 \mapsto 0101, 6 \mapsto 0110, 7 \mapsto 0111, \\ 8 \mapsto 1000, 9 \mapsto 1001 \end{array}
```

Multiples of bits

Bits seldom occur as singles. Certain multiples of bits are used as *units for information (storage) capacity*.

1 Byte: 8 bits (can represent 1 of 2⁸ = 256 alternatives). Example: one of the integer numbers between -128 and +127.

1 Halfbyte: 4 bits.

Typically, memory stores are built for *multiples of bytes*.

Prefixes: kilo, mega, giga, tera, peta, exa

- used in physics for the factors 10³, 10⁶, 10⁹, 10¹², 10¹⁵, 10¹⁸
- in computer science often used for the factors 2¹⁰, 2²⁰, 2³⁰, 2⁴⁰, 2⁵⁰, 2⁶⁰, which are slightly larger

abbre- viation	meaning	factor
KB	Kilobytes	$2^{10} = 1024$
MB	Megabytes	$2^{20} = 1,048,576$
GB	Gigabytes	2 ³⁰ = 1,073,741,824
ТВ	Terabytes	$2^{40} = 1,099,511,627,776$
PB	Petabytes	$2^{50} = 1,125,899,906,842,624$
EB	Exabytes	$2^{60} = 1,152,921,504,606,846,976$

Representation of numbers in the computer

For positive integers, basically the *binary number* system is used (cf. Part I, Chapter 6).

But: Numbers are usually stored in sections of memory of fixed size (for reasons of organization of memory access in the computer).

Integer representation in finite cells ("words" with fixed length):

Computer memory: organized in **finite cells**. Typically: Multiples of a byte.

How to store numbers in a 4-byte cell? Some encoding necessary. 2³² different values can be represented.

Example: $0 \dots 2^{32} - 1$ can be represented as binary numbers.

Example including negative numbers: $-2^{31} \dots 2^{31} - 1$ can be represented as two's complements numbers.

Two's complement: Most used representation for integers from range $-2^{n-1} \dots 2^{n-1} - 1$ (with *n*-bit cell).

Non-negative numbers: Are represented simply as binary numbers. Using n bits, the highest bit is always 0.

Negative numbers: (a) Represent their absolute value as binary number, (b) then invert all bits (including the infinite number of leading zeros, resulting in an infinite number of leading ones), and (c) add a 1. The last *n* bits are the two's complement of the value to be represented.

Example for the "Two's complement":

8-bit two's complement representation of -77

1. Represent +77 as a binary number:10011012. Invert all bits, including the leading 0s: ...1110110010

3. Add 1: ...1110110011
4. Use only the lowest (= rightmost) 8 bits: 10110011

A. Use only the lowest (= rightmost) 8 bits: 10110011 Notice:

For 16-bit cells, the result would be 11111111110110011.

decimal system	8-bit two's complement
-128	1000 0000
-127	1000 0001
-126	1000 0010
-2	1111 1110
-1	1111 1111
0	0000 0000
1	0000 0001
126	0111 1110
127	0111 1111

Properties of the two's complement:

Code represents numbers $-2^{n-1} \dots 2^{n-1} - 1$.

High bit represents sign.

Minimal value represented by 1000..., maximal by 0111....

-1 represented by 111....

Floating-point representations

Built analogously to the "scientific representation" of numbers in the form $m * 10^{e}$

- but using the binary system:

Represent numbers in the form

 $s * m * 2^e$

with sign s (+1 or -1), non-negative mantissa m, and integer exponent e.

Representation is **normalized** if $1 \le m < 2$.

Finite number of bits for sign, mantissa and exponent; often used: 32 bits (single precision), 64 bits (double precision), 80 bits (extended precision)

Typical layout of 32-bit floating point number:

```
Bit 31: represents s (1: negative; 0: positive)
```

Bits 30..23 (8 bits): represent *e*: Binary representation of e + 127, which allows the values $-126 \dots 127$. Value 0 is used in representation of number 0 and of unnormalized numbers. Value 255_{10} used to represent infinity and other exceptional values.

Bits 22..0 (23 bits): represent m, by binary representation of the integer part of $m * 2^{23}$, without the leading 1.

Example: representing +26.625 as a 32-bit normalized floating point number: $26.625_{10} = 11010.101_2$. Normalizing yields $1.1010'1010_2 * 2^4$. 32-bit floating point number (s=0, e=131_{10}):

based on representation of letters

- depending on the alphabet: certain number of bits necessary
- for 26 letters: at least 5 bits necessary $(2^4 = 16 < 26, 2^5 = 32 > 26)$
- but also encoding of digits, special signs, upper- and lower-case letters... desirable

traditional 7-bit code: ASCII (= American Standard Code for Information Interchange) ISO-646 norm later extended to 8-bit code

examples: $00110000 = hex 30 = 48_{10} = digit 0$

 $00110001 = hex 31 = 49_{10} = digit 1$... $00111010 = hex 3A = 58_{10} = ':'$... $01000001 = hex 41 = 65_{10} = 'A'$ $01000010 = hex 42 = 66_{10} = 'B'$... $01100001 = hex 61 = 97_{10} = 'a'$...

ASCII Table:

Non-printable characters						Printable characters				
Dez	Okt	Hex	Char	Code	Remark	Dez	Okt	Hex	Char	Remark
0	000	0x00	Ctrl-@	NUL	Null prompt	32	040	0x20		blank
1	001	0x01	Ctrl-A	SOH	Start of heading	33	041	0x21	!	exclamation mark
2	002	0x02	Ctrl-B	STX	Start of text	34	042	0x22	"	quotation mark
3	003	0x03	Ctrl-C	ETX	End of Text	35	043	0x23	#	
4	004	0x04	Ctrl-D	EOT	End of transmission	36	044	0x24	\$	Dollar character
5	005	0x05	Ctrl-E	ENQ	Enquiry	37	045	0x25	%	
6	006	0x06	Ctrl-F	ACK	Acknowledge	38	046	0x26	&	
7	007	0x07	Ctrl-G	BEL	Bell	39	047	0x27	'	apostroph
8	010	0x08	Ctrl-H	BS	Backspace	40	050	0x28	(
9	011	0x09	Ctrl-I	HT	Horizontal tab	41	051	0x29)	
10	012	0x0A	Ctrl-J	LF	Line feed	42	052	0x2A	*	asterisk
11	013	0x0B	Ctrl-K	VT	Vertical tab	43	053	0x2B	+	plus sign
10	014	14 0x0C Ctrl-L	G. 1 I	FF	Form feed	44	054	0x2C	,	comma
12	014	0x0C	Ctrl-L	NP	New page	45	055	0x2D	-	minus sign
13	015	0x0D	Ctrl-M	CR	Carriage return	46	056	0x2E	•	dot
14	016	0x0E	Ctrl-N	SO	Shift out	47	057	0x2F	/	slash
15	017	0x0F	Ctrl-O	SI	Shift in	48	060	0x30	0	
16	020	0x10	Ctrl-P	DLE	Data link escape	49	061	0x31	1	
17	021	0x11	Ctrl-Q	DC1	X-ON	50	062	0x32	2	
18	022	0x12	Ctrl-R	DC2		51	063	0x33	3	

19 023 0x13 Ctrl-S DC3 X-Off 20 024 0x14 Ctrl-T DC4 21 025 0x15 Ctrl-U NAK No achnowledge 22 026 0x16 Ctrl-V SYN Synchronous idle 23 027 0x17 Ctrl-W ETB End transmission blocks 24 030 0x18 Ctrl-X CAN Cancel 25 031 0x19 Ctrl-Y EM End of medium 26 032 0x1A Ctrl-Z SUB Substitute 27 033 0x1B Ctrl-Z SUB Substitute 27 033 0x1B Ctrl-Z FS File separator 28 034 0x1C Ctrl-Y FS File separator 29 035 0x1D Ctrl-Y RS Record separator 30 036 0x1E Ctrl-X RS Record separator 31 027 0x1F Ctrl-L US Unit separator 31 </th <th></th> <th></th> <th></th> <th></th> <th></th> <th></th>						
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22 026 0x16 Ctrl-V SYN Synchronous idle 23 027 0x17 Ctrl-W ETB End transmission blocks 24 030 0x18 Ctrl-X CAN Cancel 25 031 0x19 Ctrl-Y EM End of medium 26 032 0x1A Ctrl-Z SUB Substitute 27 033 0x1B Ctrl-I ESC Escape 28 034 0x1C Ctrl-I FS File separator 29 035 0x1D Ctrl-I GS Group separator 30 036 0x1E Ctrl-A RS Record separator 31 027 0x1F Ctrl-L US Unit separator	20	024	0x14	Ctrl-T	DC4	
23 027 0x17 Ctrl-W ETB End transmission blocks 24 030 0x18 Ctrl-X CAN Cancel 25 031 0x19 Ctrl-Y EM End of medium 26 032 0x1A Ctrl-Z SUB Substitute 27 033 0x1B Ctrl-I ESC Escape 28 034 0x1C Ctrl-V FS File separator 29 035 0x1D Ctrl-I GS Group separator 30 036 0x1E Ctrl-A RS Record separator 31 027 0x1F Ctrl US Unit separator	21	025	0x15	Ctrl-U	NAK	No achnowledge
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26 032 0x1A Ctrl-Z SUB Substitute 27 033 0x1B Ctrl-[ESC Escape 28 034 0x1C Ctrl-[FS File separator 29 035 0x1D Ctrl-] GS Group separator 30 036 0x1E Ctrl-^ RS Record separator 31 027 0x1F Ctrl US Unit separator	24	030	0x18	Ctrl-X	CAN	Cancel
27 033 0x1B Ctrl-[ESC Escape 28 034 0x1C Ctrl-[FS File separator 29 035 0x1D Ctrl-] GS Group separator 30 036 0x1E Ctrl-^ RS Record separator 31 027 0x1F Ctrl US Unit separator	25	031	0x19	Ctrl-Y	EM	End of medium
28 034 0x1C Ctrl-\ FS File separator 29 035 0x1D Ctrl-] GS Group separator 30 036 0x1E Ctrl-^ RS Record separator 31 027 0x1F Ctrl US Unit separator	26	032	0x1A	Ctrl-Z	SUB	Substitute
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30 036 0x1E Ctrl-^ RS Record separator 31 027 0x1F Ctrl US Unit separator	28	034	0x1C	Ctrl-\	FS	File separator
31 027 0x1F Ctrl US Unit separator	29	035	0x1D	Ctrl-]	GS	Group separator
	30	036	0x1E	Ctrl-^	RS	Record separator
127 0177 0x7F DEL Delete or rubout	31	027	0x1F	Ctrl	US	Unit separator
	127	0177	0x7F		DEL	Delete or rubout

52	064	0x34	4		
53	065	0x35	5		
54	066	0x36	6		
55	067	0x37	7		
56	070	0x38	8		
57	071	0x39	9		
58	072	0x3A	:	colon	
59	073	0x3B	;	semicolon	
60	074	0x3C	<	less than	
61	075	0x3D	=	euqality character	
62	076	0x3E	>	greater than	
63	077	0x3F	?	interrogation mark	
64	0100	0x40	@	at	
65	0101	0x41	A		
66	0102	0x42	В		
67	0103	0x43	C		
68	0104	0x44	D		
69	0105	0x45	E		
70	0106	0x46	F		
71	0107	0x47	G		
72	0110	0x48	Н		
73	0111	0x49	Ι		
74	0112	0x4A	J		
75	0113	0x4B	K		
76	0114	0x4C	L		
77	0115	0x4D	M		
78	0116	0x4E	N		
79	0117	0x4F	0		
80	0120	0x50	Р		
81	0121	0x51	Q		
82	0122	0x52	R		
83	0123	0x53	S		
84	0124	0x54	Т		
85	0125	0x55	U		
86	0126	0x56	V		
87	0127	0x57	W		
88	0130	0x58	X		
89	0131	0x59	Y		
90	0132	0x5A	Z		
91	0133	0x5B	[
92	0134	0x5C		backslash	
93	0135	0x5D]		
94	0136	0x5E	^	caret	
95	0137	0x5F	_	low line	
96	0140	0x60	`	back quote	
97	0141	0x61	a		
98	0142	0x62	b		
99	0143	0x63	c		
100	0144	0x64	d		
101	0145	0x65	e		
102	0146	0x66	f		
					1

103 0147 0x67 g
104 0150 0x68 h
105 0151 0x69 i
106 0152 0x6A j
107 0153 0x6B k
108 0154 0x6C 1
109 0155 0x6D m
110 0156 0x6E n
111 0157 0x6F o
112 0160 0x70 p
113 0161 0x71 q
114 0162 0x72 r
115 0163 0x73 s
116 0164 0x74 t
117 0165 0x75 u
118 0166 0x76 v
119 0167 0x77 w
120 0170 0x78 x
121 0171 0x79 y
122 0172 0x7A z
123 0173 0x7B {
124 0174 0x7C
125 0175 0x7D }
126 0176 0x7E ~

ASCII not sufficient for alphabets of the non-Angloamerican world (not even for European alphabets with ä, ö, ü, ß, é, ø, ñ, å...)

Unicode:

2 byte (= 16 bit) code for multilingual text processing - can represent 65536 characters

amongst them: 27786 Chinese-Japanese-Korean characters 11172 Hangul characters (Korean) ancient Nordic runes Tibetan characters Cherokee characters ...

complete list see http://www.unicode.org/charts/

Unicode "Escape sequence" (to utilise it in the programming language Java):

e.g., $\0041 = A'$ (0041 = hexadecimal representation)

Some characters occur more frequently in texts than others:

better use variable-length code

UTF-8: Universal Transformation Format Characters encoded with variable number of bytes \Rightarrow for texts with many ASCII characters (like on many web pages) shorter as Unicode

Strings (or *words*): sequences of characters encoded by sequences of the corresponding code words

Digital representation of pictures

Gray levels: encode each gray level by a number from a fixed interval (e.g. 0, 1, ..., 255: 8-bit representation - 0 = black, 255 = white)

Colours: several colour models possible the most frequently used one: *RGB model* (red / green / blue: primary colours for additive colour composition) Each colour from a certain range ("gamut") can be mixed from these primary colours

examples with 8-bit intensities:

black	(0, 0, 0)
white	(255, 255, 255)
medium gray	(127, 127, 127)
red	(255, 0, 0)
green	(0, 255, 0)
blue	(0, 0, 255)
light blue	(127, 127, 255)
yellow	(255, 255, 0)

Pictures:

typically represented as raster images – rectangular array (matrix) of *pixels*, each pixel represented by its 3 colour values.

Representation of text documents (book pages, web pages...)

Level of representation is important.

- (1) Is there text on the page? One bit.
- (2) What is the text on the page? –
 Representation of letter sequence (e.g., string of ASCII characters).
- (3) What is the exact layout of the text on the page? "formatted text"
 - use special characters for formatting, or
 - represent the page by a rasterized black-and-white image.

Text documents with graphical elements:

- represent all as a single raster image, or

 use combined representation: several data files, one for the text, the other for the pictorial parts
 → HTML web pages are built like this

file <name>.html or <name>.htm contains text, layout information and links to other pages files <name>.gif or <name>.jpg or <name>.png contain images

Messages and redundancy

Message: A finite sequence of letters, used to transfer some information via encoding/transfer/decoding

Signal: The physical representation of the message (examples: as voltage pattern or light pattern)

Redundancy: Part of a message which is not necessary for the transferred information (later explained more exactly)

Error correction by **redundant** codes: Natural languages allow to detect many errors.

Example in informatics: **Parity bits**. Even parity: 9 bits per byte. 9th bit makes number of one-bits even. Allows detection of single-bit errors. Computer memory sometimes uses 9 bits per byte for this purpose.

Other example: ISBN code (International Standard Book Number) – last character is a parity character

Entropy and quantification of information

Shannon's information theory:

Information as a measurable, statistical property of signals

How can we measure information and redundancy of characters in a message?

Assumption: *N*-character alphabet { $x_1, x_2, ..., x_N$ }

Number of bits per character:

$$H_0 = \log_2 N$$

(Remember: $\log_2 N = (\log N)/(\log 2)$)

Information content of a single character x_i : $\log_2 \frac{1}{p(x_i)}$

Here, $p(x_i)$ is the probability of x_i .

Entropy = average value of information content of all characters = $H = \sum_{k=1}^{N} p(x_k) * \log_2 \frac{1}{1}$

$$= H = \sum_{k=1}^{k} p(x_k) * \log_2 \frac{1}{p(x_k)}$$

Binary encoding needs at least, on average, *H* bits per character.

Redundancy: $R = H_0 - H$.

Example: Four-letter alphabet $\{a, b, c, d\}$

Probabilities: $p_a = 0.5, p_b = 0.25, p_c = 0.125, p_d = 0.125$

Thus:

 $H_0 = 2$ bits per character encodable

Entropy: 0.5 * 1 + 0.25 * 2 + 0.125 * 3 + 0.125 * 3 = 1.75 bits per character encoded

Redundancy: 0.25 bits per character

Examples:

– $a\mapsto$ 00, $b\mapsto$ 01, $c\mapsto$ 10, $d\mapsto$ 11: on average 2 bits per character

 $-a \mapsto 0, b \mapsto 10, c \mapsto 110, d \mapsto 111$: on average 1.75 bits per character (optimal, no redundancy)