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Computer Science and Mathematics, summer term 2016 Test exam

	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Σ
max.	7	13	5	5	5	5	10	10	60
credits									

Please, read all tasks carefully.

Write down the way how you got your result; credits are given also for the right approach towards a solution.

Time for this exam: 90 min.; no electronic devices are allowed.

- 4 pages in total -

Task 1 (Linear algebra: vectors)

Three vectors in \mathbb{R}^3 are given:

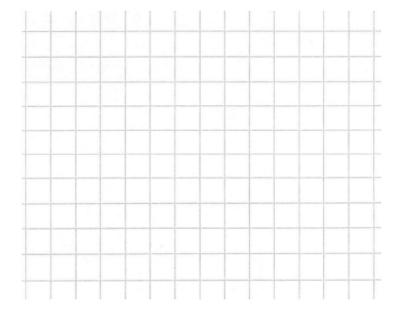
$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}, \ \vec{c} = \begin{pmatrix} -16 \\ 0 \\ -10 \end{pmatrix}.$$

- (a) Calculate the vector $2 \cdot \vec{a} + \vec{b}$. (2 cr.)
- (b) Calculate the inner product $\vec{a} \cdot \vec{b}$. (1 cr.)
- (c) What is the angle between \vec{a} and \vec{b} ? (1 cr.)
- (d) Are the three vectors \vec{a} , \vec{b} , \vec{c} linearly independent? Prove your answer. (2 cr.)
- (e) Give a geometrical description of the shape of the set of points in space described by { $\vec{x} \in \mathbb{R}^3 | \vec{c} \cdot \vec{x} = 0$ } ("." denotes the inner product of vectors.) (1 cr.)

Task 2 (Linear algebra: 2 × 2 matrices and linear mappings)

The matrix $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is given.

- (a) What is the rank of A? Give a reason for your answer.
- (b) Calculate $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Draw the vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}, A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ in a cartesian coordinate system. (4 cr.)



(c) The linear mapping associated with A is $f: \vec{x} \mapsto A \cdot \vec{x}$. Describe (in words) how an arbitrary vector \vec{x} is transformed geometrically by f. (1 cr.)

(d) Calculate the matrix A^2 .	(1 cr.)
(e) Calculate det A.	(1 cr.)

- (f) Determine the matrix A^{-1} (if it exists).
- (g) Determine the eigenvalues of A. (3 cr.)

(2 cr.)

(1 cr.)

Task 3 (Linear algebra: larger matrices and linear systems)

The matrix
$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 6 \end{pmatrix}$$
 is given.
(a) Calculate the product $A \cdot \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix}$ (2 cr.)
(b) $A\vec{x} = \vec{b}$ with $\vec{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a system of $m = 3$ linear equations for $n = 3$ unknowns.

How many unknowns can be chosen arbitrarily? (Give a reason for your answer.) (3 cr.)

Task 4 (Computer science: programming)

 $\left(2 \right)$

(5 cr.)

The following Java method **f** gets an integer array **x** and a single integer **a** as its arguments:

```
public int f(int x[], int a)
   {
   int i = 0;
   boolean b = true;
   while (b && (i < x.length))
      ł
      if (x[i] == a)
         b = false;
      else
         i = i+1;
      }
   if (b)
      ł
      println("Error!");
      return -1;
      }
   else
      return i;
   }
```

What does it give back as its result?

Task 5 (Computer science: representation of numbers)

(a) What is the binary representation of the decimal number 63 ?	(1 cr.)
(b) What is the decimal representation of the hexadecimal number 2A5 ?	(1 cr.)
(c) Give the 8-bit two's complement of the decimal number –84.	(2 cr.)

(d) What is the exact value of the fraction which is represented in the ternary system (= base 3) by 0.2222222... (with infinitely many 2s after the dot) ? (1 cr.)

Task 6 (Computer science: rule-based simulation)

(a) Given is the L-system

L1: $A \rightarrow [RU(45) F0] F0 A$.

Which string is produced after 3 steps of application of this rule to the start word *A* ? Draw the graphical structure in the plane which is obtained from this string by turtle interpretation. (2 cr.)

(b) Now this is modified to the L-system L2: $A \rightarrow [RU(45) FO A] FO A$.

Draw the corresponding graphical structure after 3 steps in this case. (1 cr.)

(c) How does the number of single lines (obtained from an "F0" symbol) grow (quantitatively) with the number of steps for L1, how for L2 ? (2 cr.)

Task 7 (Calculus: univariate functions, differentiation)(10 cr.)

Given is the following function: $f(x) = \frac{1}{3}x^3 - 4x^2 + 7x - 5$.

- (a) Find all x values where the function f has local extrema and classify them as minima or maxima.
- (b) Find where the function is increasing / decreasing, and all x values of inflection points.

Task 8 (Calculus: integration)(10 cr.)

Compute the total area between the function $f(x) = 6x^2 + 6x - 12$, the x axis and the lines $x_1 = 0$ and $x_2 = 2$.