

Computer Science and Mathematics, summer term 2016
Test exam

	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Σ
max.	7	13	5	5	5	5	10	10	60
credits									

Please, read all tasks carefully.

Write down the way how you got your result; credits are given also for the right approach towards a solution.

Time for this exam: 90 min.; no electronic devices are allowed.

- 4 pages in total -

Task 1 (Linear algebra: vectors)

Three vectors in \mathbb{R}^3 are given:

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \vec{b} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}, \vec{c} = \begin{pmatrix} -16 \\ 0 \\ -10 \end{pmatrix}.$$

- (a) Calculate the vector $2 \cdot \vec{a} + \vec{b}$. (2 cr.)
- (b) Calculate the inner product $\vec{a} \cdot \vec{b}$. (1 cr.)
- (c) What is the angle between \vec{a} and \vec{b} ? (1 cr.)
- (d) Are the three vectors \vec{a} , \vec{b} , \vec{c} linearly independent? Prove your answer. (2 cr.)
- (e) Give a geometrical description of the shape of the set of points in space described by
 $\{ \vec{x} \in \mathbb{R}^3 \mid \vec{c} \cdot \vec{x} = 0 \}$
 ("." denotes the inner product of vectors.) (1 cr.)

Task 2 (Linear algebra: 2×2 matrices and linear mappings)

The matrix $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is given.

(a) What is the rank of A ? Give a reason for your answer. (1 cr.)

(b) Calculate $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Draw the vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ in a cartesian coordinate system. (4 cr.)



(c) The linear mapping associated with A is $f: \vec{x} \mapsto A \cdot \vec{x}$. Describe (in words) how an arbitrary vector \vec{x} is transformed geometrically by f . (1 cr.)

(d) Calculate the matrix A^2 . (1 cr.)

(e) Calculate $\det A$. (1 cr.)

(f) Determine the matrix A^{-1} (if it exists). (2 cr.)

(g) Determine the eigenvalues of A . (3 cr.)

Task 3 (Linear algebra: larger matrices and linear systems)

The matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 4 & 2 & 6 \end{pmatrix}$ is given.

(a) Calculate the product $A \cdot \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ -1 & 0 \end{pmatrix}$ (2 cr.)

(b) $A\vec{x} = \vec{b}$ with $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ is a system of $m = 3$ linear equations for $n = 3$ unknowns.

How many unknowns can be chosen arbitrarily? (Give a reason for your answer.) (3 cr.)

Task 4 (Computer science: programming) (5 cr.)

The following Java method `f` gets an integer array `x` and a single integer `a` as its arguments:

```
public int f(int x[], int a)
{
    int i = 0;
    boolean b = true;
    while (b && (i < x.length))
    {
        if (x[i] == a)
            b = false;
        else
            i = i+1;
    }
    if (b)
    {
        println("Error!");
        return -1;
    }
    else
        return i;
}
```

What does it give back as its result?

Task 5 (Computer science: representation of numbers)

- (a) What is the binary representation of the decimal number 63 ? (1 cr.)
- (b) What is the decimal representation of the hexadecimal number 2A5 ? (1 cr.)
- (c) Give the 8-bit two's complement of the decimal number -84. (2 cr.)
- (d) What is the exact value of the fraction which is represented in the ternary system (= base 3) by 0.222222... (with infinitely many 2s after the dot) ? (1 cr.)

Task 6 (Computer science: rule-based simulation)

- (a) Given is the L-system

L1: $A \rightarrow [RU(45)F0]F0A$.

Which string is produced after 3 steps of application of this rule to the start word A ?

Draw the graphical structure in the plane which is obtained from this string by turtle interpretation. (2 cr.)

- (b) Now this is modified to the L-system

L2: $A \rightarrow [RU(45)F0A]F0A$.

Draw the corresponding graphical structure after 3 steps in this case. (1 cr.)

- (c) How does the number of single lines (obtained from an "F0" symbol) grow (quantitatively) with the number of steps for L1, how for L2 ? (2 cr.)

Task 7 (Calculus: univariate functions, differentiation) (10 cr.)

Given is the following function: $f(x) = \frac{1}{3}x^3 - 4x^2 + 7x - 5$.

- (a) Find all x values where the function f has local extrema and classify them as minima or maxima.
- (b) Find where the function is increasing / decreasing, and all x values of inflection points.

Task 8 (Calculus: integration) (10 cr.)

Compute the total area between the function $f(x) = 6x^2 + 6x - 12$, the x axis and the lines $x_1 = 0$ and $x_2 = 2$.