Exercises (Combinatorics, computer science)

Task 10 *Combinatorics*

Permutations can be defined as words of length n over an alphabet with n elements, with the restriction that no letter appears twice in the word.

(a) List systematically all permutations of the $n = 4$ letters A, B, C, D.

(b) Prove for the number $P(n)$ of permutations of n letters:

$$P(n) = n \cdot P(n-1) \quad \text{if } n > 1.$$

Task 11 *Graphs*

List all undirected, non-isomorphic graphs with 4 vertices and 3 edges. (Multiple edges and loops are not allowed.) How many of them are connected, how many are not?

Task 12 *Digital storage requirements*

Which data needs more storage capacity:

(a) a DNA sequence of 8 million nucleotides (each being represented as A, T, G or C and requiring 2 bits), or

(b) a colour image, consisting of $1024 \cdot 1024$ pixels, where each pixel carries 24 bits of colour information?

Prove your answer by evaluating both storage requirements in terms of MB (MegaBytes) and comparing them.

Task 13 *Programming*

(a) Write a mathematical formula which gives the meaning of the Java expression
`x + Math.sqrt(1-x*x)/2`.

(b) What kind of runtime error can easily occur when the above expression is evaluated in a Java programme? Which mathematical condition (for **`x`**) has to be fulfilled to prevent this error?

Task 14 *Rule-based simulation*

Draw the geometrical structure which is generated after 2 steps of rule application by the following L-system (or XL code), assuming the 3-d object definitions

```
module Bud extends Sphere(1.0);    /* 1.0 is radius */
module Shoot extends F(5.0, 1.0);  /* 5.0 is length of
                                      cylinder, 1.0 is diameter */
```

are in force and the start word is **`Bud`**:

```
Bud ==> Shoot [ RU(45) Bud ] [ RU(-45) Bud ];
```