Computer Science and Mathematics
Summer term 2015

Exercises 4

1. Evaluate the following determinants:

(a) $\begin{vmatrix} a+b & 2b \\ 2a & a+b \end{vmatrix}$

(b) $\begin{vmatrix} 2 & 0 & 8 & 1 \\ 5 & 3 & 1 & 6 \\ 7 & 0 & 2 & 0 \\ 1 & 0 & 0 & -1 \end{vmatrix}$

2. For which of the following pairs of matrices is their product defined?
In these cases, calculate the resulting matrix.

(a) $\begin{pmatrix} 4 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 \\ 3 & x \end{pmatrix}$

(b) $\begin{pmatrix} 5 & 3 \\ 1 & 0 \\ 8 & 7 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 0 & -6 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 10 \\ 0 & 2 & 0 \\ 1 & 5 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 30 \\ 20 \\ 10 \end{pmatrix}$

(e) $\begin{pmatrix} 50 \\ 60 \end{pmatrix} \cdot \begin{pmatrix} -2 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

3. A tree stand at time *t* consists of 20 ha of young trees (age class 1), 40 ha of medium-age trees (class 2) and 40 ha of old trees (class 3).
After 1 time step, the following events have occurred:

- 25 % of the young trees have been killed by the Dark Monster Beetle,
- 10 % of the medium-age trees have been lethally damaged by the Wild Crazy Deer,
- all old trees have been harvested,
- all damaged trees have been removed,
- all open spaces have been reforested with young trees,
- all other trees have grown older into the next age class.

Give the transition matrix of this age-class dynamics and calculate the age-class vector of the stand at time t+1.

4. Do the following matrices have an inverse? If so, calculate it.

(a) $A = \begin{pmatrix} 6 & -10 \\ -9 & 15 \end{pmatrix}$

(b) $B = \begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix}$

5. Decide with Frobenius' theorem if the following system of linear equations has a unique solution:

$$\begin{aligned} x - 5y &= 4z \\ x - 2y + 2z &= 1 \\ 3y - z &= 15 \end{aligned}$$

6. Determine the eigenvalues of the matrix $A = \begin{pmatrix} 5 & 7 \\ 3 & 1 \end{pmatrix}$.