Computer Science and Mathematics
Summer term 2015

Exercises 2

1. The following sets are sets of number pairs, i.e., subsets of the cartesian product $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$:

   $A := \{ (x, y) \in \mathbb{R}^2 \mid y = \frac{2}{3}x - 2 \}$
   $B := \{ (x, y) \in \mathbb{R}^2 \mid y = -|0.5x| + 1 \}$
   $C := \{ (x, y) \in \mathbb{R}^2 \mid x \geq 0 \wedge -x+3 \geq y \geq 0 \}$

   (a) Visualize each of the sets A, B, C in the cartesian coordinate system. (Make a separate graphical image for each set.)
   (b) All these sets are relations. Which of them are even functions?

2. Let P be the set of all participants of a party and D the set of all drinks which are offered there. Each participant $p_i \in P$ shall get a drink $d_i \in D$ according to his/her preference. In this way, a mapping $f: P \to D$ shall be defined.
   What do the following possible properties of f mean in this context?
   (a) surjectivity
   (b) injectivity
   (c) bijectivity
   (d) Can a participant of the party get two different drinks?

3. Let $f : D \to R$ be the function described by $f(x) = 2x^3 - 1$.
   The domain D is defined as $D := \{ x \in \mathbb{R} \mid -3 \leq x < 2 \}$. Determine
   (a) the range $R := f(D)$,
   (b) a formula for the inverse function f^{-1} .

4. (a) Calculate the decimal value of the binary number 1001111.
   (b) Calculate the hexadecimal representation of the decimal number 999.
   (c) What is the binary expansion of the value 1/3 ?
   (Hint: You can do "written division" analogously to the decimal case, but with doubling the remainder in every step instead of multiplying by 10.)

5. Let $\vec{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

   (a) Draw $\vec{a}$ and $\vec{b}$ in a coordinate system.
   (b) Determine $2 \cdot \vec{a} + \vec{b}$ by calculation and graphically.
   (c) Let $\vec{c} = \begin{pmatrix} 1 \\ -11 \end{pmatrix}$. Find a representation of $\vec{c}$ as a linear combination of $\vec{a}$ and $\vec{b}$.

6. Let $\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. What geometrical objects are described by the following sets?

   (a) $\{ \vec{b} + t \cdot \vec{a} \mid t \in \mathbb{R} \wedge t \geq 0 \}$
   (b) $\{ \vec{x} \in \mathbb{R}^2 \mid \vec{a} \cdot \vec{x} = 0 \}$
   (c) $\{ \vec{x} \in \mathbb{R}^2 \mid \|\vec{x} - \vec{b}\| = 0.5 \}$

7. (a) Are the vectors $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -3 \\ -2 \\ 5 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$ linearly independent?

   (b) What is the maximal number of vectors which can be linearly independent in $\mathbb{R}^4$?

8. The points $A = (1; 3)$, $B = (11; 7)$ and $C = (3; 13)$ are given in the cartesian coordinate system.

   (a) Let A be the new zero (origin) and calculate the vectors $\vec{b} = \overrightarrow{AB}$ and $\vec{c} = \overrightarrow{AC}$.

   (b) Calculate the vector $\vec{d} = \vec{b} + \vec{c} = \overrightarrow{AD}$ and the absolute coordinates of the new point D.

   (c) Calculate the inner product $\vec{b} \cdot \vec{c}$ and the angle $\angle(\vec{b}, \vec{c})$.

   (d) Extend the vectors by a third dimension (with value 0) and calculate the cross product $\vec{b} \times \vec{c}$.

   (e) Calculate the area of the parallelogram spanned by $\vec{b}$ and $\vec{c}$.