Exercises Part 13 with Solutions

1. In some study the diameter of potatoes for chips production was investigated. It must be calculated, what fraction of potatoes has the diameter between 5 and 6 cm, because this size of potatoes is the most suitable for chips production. The sample of 1000 potatoes was examined. The mean was 5.3 cm, the standard deviation was 0.5 cm. We can suppose that they are equal to the mean μ and to standard deviation σ of the population. What fraction of potatoes has the diameter between 5 and 6 cm? The diameter is approximately normal distributed.

Solution.

X = diameter of potatoes; $\mu = 5.3$; $\sigma_x = 0.5$.

$P(5 \leq X \leq 6) = 1 - P(X < 5) - P(X > 6)$

$$z_1 = \frac{(5 - 5.3)}{0.5} = -0.6;$$
$P(X < 5) = P(Z < z_1 = -0.6) = P(Z > -z_1 = 0.6) = 0.2743$

$$z_2 = \frac{(6 - 5.3)}{0.5} = 1.4;$$
$P(X > 6) = P(Z > z_2 = 1.4) = 0.0808$

$\Rightarrow$
$P(5 \leq X \leq 6) = 1 - P(X < 5) - P(X > 6) = 1 - 0.2743 - 0.0808 = 0.6449$

64.49% of potatoes have the diameter between 5 and 6 cm.

2. Two systems of production of sorghum **A** and **B** was examined on many farms in Sahel. The sorghum yield was measured on one field (in dt/ha) of every farm. The mean was 25 dt/ha and the standard deviation was 3.5 dt/ha. The histogram has shown, that the data was approximately normal distributed. How high is the probability, that a yield of sorghum is less, than 20 dt/ha? How high is this probability of this low values by the system **B** with the mean of 30 dt/ha and the standard deviation of 4.2 dt/ha.

Solution

System **A**: X = sorghum yield on one field (dt/ha); $\mu = 25$; $\sigma_x = 3.5$.

Find:

$P(X < 20)$;

$$z = \frac{(20 - 25)}{3.5} = -1.43$$

$$P(X < 20) = P(Z < z) = -1.43 = P(Z > -z = 1.43) = 0.0764$$

The probability, that a yield of sorghum is less, than 20 dt/ha is 7.64%.

System **B**: X = sorghum yield on one field (dt/ha); $\mu = 30$; $\sigma_x = 4.2$.

$P(X < 20)$;

$$z = \frac{(20 - 30)}{4.2} = -2.38$$

$$P(X < 20) = P(Z < z) = -2.38 = P(Z > -z = 2.38) = 0.0087$$

The probability, that a yield of sorghum is less, than 20 dt/ha is 0.87%.

3. Given is the herd of 200.000 cows. The mean body weight amounts to $\mu = 650$ kg and the standard deviation of population is $\sigma = 25\ kg$. The body weight is approximately normal distributed. One sample of the size of $n = 100$ was taken from this population.

a. Which fraction of individual values do you expect between 645 and 655 kg?

b. Which fraction of sample mean values do you expect between 645 and 655 kg?

Solution:

a. Individual values are approximately normal distributed with the mean (Expected value) $\mu = 650$ and standard deviation $\sigma_x = 25$.

Find: $P(645 \leq X \leq 655)$.

$P(645 \leq X \leq 655) = 1 - P(X < 645) - P(X > 655)$

$$z_1 = \frac{(645 - 650)}{25} = -0.2;$$

$P(X < 645) = P(Z < z_1 = -0.2) = P(Z > -z_1 = 0.2) = 0.4207$

$$z_2 = \frac{(655 - 650)}{25} = 0.2;$$

$P(X > 655) = P(Z > z_2 = 0.2) = 0.4207$

$$\Rightarrow P(645 \leq X \leq 655) = 1 - P(X < 645) - P(X > 655) = 1 - 0.4207 - 0.4207 \\ = 0.1586$$

We expect 15.86% of individual values between 645 and 655 kg. One individual value lies with 15.86% probability between 645 and 655 kg.

b. Sample means are approximately normal distributed with the mean (Expected value) $\mu = 650$ and standard error $\sigma_{\bar{x}} = \sigma_x/\sqrt{n} = \frac{25}{10} = 2.5$.
Find: $P(645 \leq X \leq 655)$.

$$P(645 \leq X \leq 655) = 1 - P(X < 645) - P(X > 655)$$

$$z_1 = \frac{(645 - 650)}{2.5} = -2;$$
$$P(\bar{X} < 645) = P(Z < z_1 = -2) = P(Z > -z_1 = 2) = 0.0228$$

$$z_2 = \frac{(655 - 650)}{2.5} = 2;$$
$$P(\bar{X} > 655) = P(Z > z_2 = 2) = 0.0228$$

$$\Rightarrow P(645 \leq \bar{X} \leq 655) = 1 - P(\bar{X} < 645) - P(\bar{X} > 655) = 1 - 0.0228 - 0.0228 = 0.9544$$

We expect 95.44% of sample means $\bar{X}$ between 645 and 655 kg.

4. Suppose that the sample size was increased from $n = 100$ to $n = 130$ cows.

a. What fraction of individual values do you expect between 645 and 655 kg?

b. What fraction of sample mean values do you expect between 645 and 655 kg?

c. Compute a 95% confidence interval for the mean weight.

Solution:

a. The distribution of the individual values does not change through change of the sample size n! $\Rightarrow$ The same answer, as in the task 3a.

b. Sample means are approximately normal distributed with the mean (Expected value) $\mu = 650$ and standard error $\sigma_{\bar{x}} = \sigma_x/\sqrt{n} = \frac{25}{\sqrt{130}} = 2.193$.

Find: $P(645 \leq \bar{X} \leq 655)$.

$P(645 \leq \bar{X} \leq 655) = 1 - P(\bar{X} < 645) - P(\bar{X} > 655)$

$$z_1 = \frac{(645 - 650)}{2.193} = -2.28;$$

$P(\bar{X} < 645) = P(Z < z_1 = -2.28) = P(Z > -z_1 = 2.28) = 0.0113$

$$z_2 = \frac{(655 - 650)}{2.193} = 2.28;$$

$P(\bar{X} > 655) = P(Z > z_2 = 2) = 0.0113$

$\Longrightarrow P(645 \leq \bar{X} \leq 655) = 1 - P(\bar{X} < 645) - P(\bar{X} > 655) = 1 - 0.0113 - 0.0113$
$= 0.9774$

We expect 97.74% of sample means $\bar{X}$ between 645 and 655 kg.

c. 95% confidence interval for the mean weight:

$$lower\ limit: \bar{x} - z_{1-\alpha/2} \cdot \sigma_{\bar{x}} = 650 - 1.96 \cdot 2.193 = 650 - 4.3 = 645.7$$

$$upper\ limit: \bar{x} + z_{1-\alpha/2} \cdot \sigma_{\bar{x}} = 650 - 1.96 \cdot 2.193 = 650 + 4.3 = 654.3$$

With the probability of 95% the the mean weight lies between $645.7$and 654.3 kg.

5. You have to prepare a solution with pH-value equal to 2.0. The pH-value of the produced solution is 2.27. The accuracy of measurement is given by the standard deviation $\sigma = 0.1$. The measured values are approximately normal distributed. How high is the probability to reach such a deviation or more high deviation between the theoretical value and the measured value. Attention: You have to pay attention to deviations in both directions (two-tailed problem).

Solution

$\mu = 2.0$; $\sigma_x = 0.1$; measured: $x = 2.27$;

Deviation from expected value is 0.27

Find: $P(X > 2.27)$ and $P(X < 1.73)$

$$z_1 = \frac{(2.27 - 2.00)}{0.1} = 2.7;$$

$$P(X > 2.27) = P(Z > 2.7) = 0.0035$$

$$z_2 = \frac{(1.73 - 2.00)}{0.1} = -2.7$$

$$P(X < 1.73) = P(Z < -2.7) = P(Z > 2.7) = 0.0035$$

The probability to reach such a deviation or higher deviation between the theoretical value and the measured value is

$$\Longrightarrow P(X > 2.27) + P(X < 1.73) = P(X \geq 2.27) + P(X \leq 1.73) = 2 \cdot 0.0035$$
$$= 0.0070 = 0.7\%$$

The exeedance probability is here very small, so there are no plausible reasons to think, that the true pH value is 2.0.

6. Some firm produces buns. The buns weight x is approximately normal distributed. Standard deviation is: $s = 1.2$ g. The buns are delivered to the buns seller with the notice, that the average weight of the buns is 50 g. Formulate the null hypotheses and answer the question, if the measured average weight of 25 buns $\bar{x} = 49.0$ differs significantly from the theoretical weight = 50 g.

Solution

$H_0: \mu = 50$

$H_1: \mu \neq 50$

$$z_{Test} = \frac{\bar{x} - \mu}{\sqrt{s_x^2/n}} = \frac{1.0}{0.24} = 4.17$$

$z_{Tab} = 1.96$

$z_{Test} > z_{Tab} \Rightarrow H_0$ is rejected

The average weight of 25 buns $\bar{x} = 49.0$ differs significantly from the theoretical weight = 50 g.

7. One field trial with some plant was carried out on 8 fields. The measured yields were as follow (in dt/ha):

20	22	26	24	21	27	24	25

The firm will produce seeds of this plant, if the average yield will be higher, than 22 dt/ha.
Test with significant level $\alpha = 0.05$, if the measured yields are higher, than 22 dt/ha.

Solution

$H_0: \mu \leq 22$
$H_0: \mu > 22$

One-tailed test

$$z_{Test} = \frac{\bar{x} - \mu_0}{\sqrt{s_x^2/n}}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{189}{8} = 23.625$$

$$s_x^2 = \frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1} = \frac{4507 - \frac{35721}{8}}{7} = 5.98$$

$$z_{Test} = \frac{\bar{x} - \mu_0}{\sqrt{s_x^2/n}} = \frac{23.625 - 22}{\sqrt{5.98/8}} = 1.88$$

$z_{Test} > z_{Tab} = 1.64 \Rightarrow H_0$ is rejected

The measured yields are significantly higher, than 22 dt/ha.