Exercises Part 12 with Solutions

1. Which of these statements are true?

Solution:

- Calculating of means by metric data doesn't make sense **-**
- Ordinal data are rank-ordered data **+**
- By nominal data no rank-ordered categories exist **+**
- Nominal data are representable on the number line **-**
- Differences of ordinal data are meaningful **-**
- Metric data are always continuous **-**

2. Given are the weights of cattle in the herd in kg ($n = 40$ Animals).

295 248 260 223 306 234 263 248
235 251 232 244 267 230 231 230
259 242 216 261 283 286 240 290
224 226 274 260 248 281 234 234
241 248 274 220 280 241 277 247

Present the data as a histogram.

Solution:

Sturge's formula: $k = 1 + 3.32log_{10}(n) = 1 + 3.32log_{10}(40) = 6.32$

Terrel-Scott formula: $k \geq (2n)^{\frac{1}{3}} = (80)^{\frac{1}{3}} = 4.3$

Choose $k = 5$

$x_{min} = 216, x_{max} = 306$

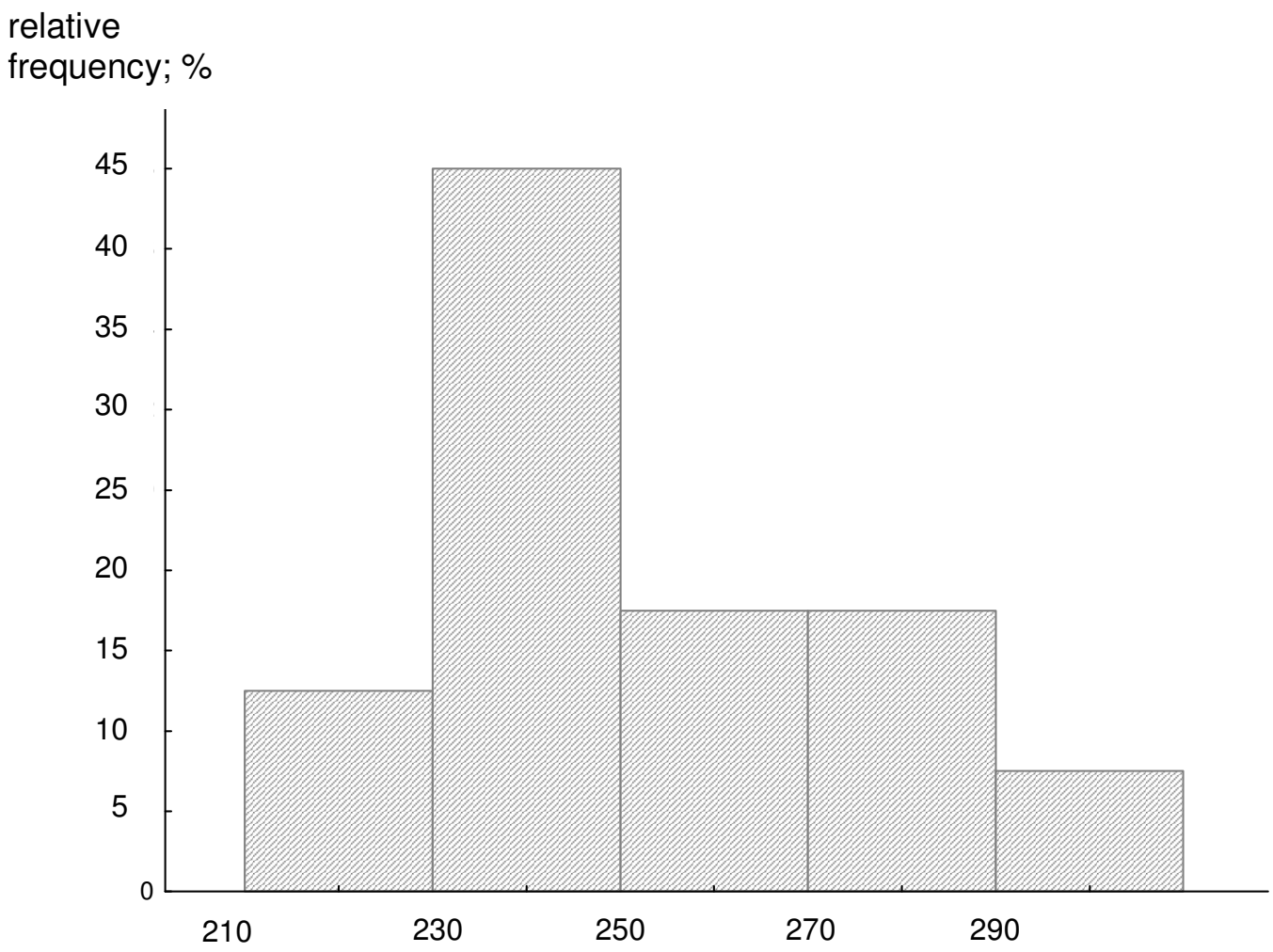
$V = 306 - 216 = 90,$

$b > \frac{V}{k} = \frac{90}{5} = 18$

Choose $b = 20$

weight (kg)	Number of animals/ absolute frequency	percent/ relative frequency	[$]Height of bars of histogram (cm) (recommendation)
210 – 229	5	12.5	2.5
230 – 249	18	45.0	9.0
250 – 269	7	17.5	3.5
270 – 289	7	17.5	3.5
290 – 309	3	7.5	1.5

[$] Height = relative frequency (%) $\times$ 2/10


relative
frequency; %
45
40
35
30
25
20
15
10
5
0
210
230
250
270
290

3. Given are the weights of cattle in a small herd in kg ($n = 12$ animals).

295 248 260 223 306 234 263 248 235 251 232 244

Calculate the Quantiles Q_0, Q_{25}, Q_{50}, Q_{75}, Q_{100}.

Calculate the median and the mean.

Calculate the variance, the standard deviation and the coefficient of variation.

Calculate the range and interquartile range.

Present the results as a Box-Plot.

Solution:

The ascending sequence of sorted values: $n = 12$

223	232	234	235	244	248	248	251	260	263	295	306
$x_{[1]}$	$x_{[2]}$	$x_{[3]}$	$x_{[4]}$	$x_{[5]}$	$x_{[6]}$	$x_{[7]}$	$x_{[8]}$	$x_{[9]}$	$x_{[10]}$	$x_{[11]}$	$x_{[12]}$

$$Q_0 = 223$$

$$Q_{25} = \frac{234 + 235}{2} = 234.5$$

$$Q_{50} = \frac{248 + 248}{2} = 248$$

$$Q_{75} = \frac{260 + 263}{2} = 234.5$$

$$Q_{100} = 306$$

Table

i	x_i	x_i^2
1	295	87025
2	248	61504
3	260	67600
4	223	49729
5	306	93636
6	234	54756
7	263	69169
8	248	61504
9	235	55225
10	251	63001
11	232	53824
12	244	59536
$\sum_{i=1}^{n}$	3039	776509

Mean

$$\bar{x} = \frac{\sum_{i=1}^{12} x_i}{12} = \frac{3039}{12} = 253.25$$

Variance:

$$s^2 = \frac{\sum_{i=1}^{12} x_i^2 - \frac{\sum_{i=1}^{12} x_i}{n}}{n-1} = \frac{776509 - \frac{3039}{12}}{11} = \frac{776509 - \frac{9235521}{12}}{11} = \frac{6882.25}{11} = 625.66$$

Standard deviation:

$$s = \sqrt{625.66} = 25.01$$

Coefficient of variation

$$CV = \frac{s}{\bar{x}} = \frac{25.01}{253.25} \cdot 100\% = 9.88\%$$

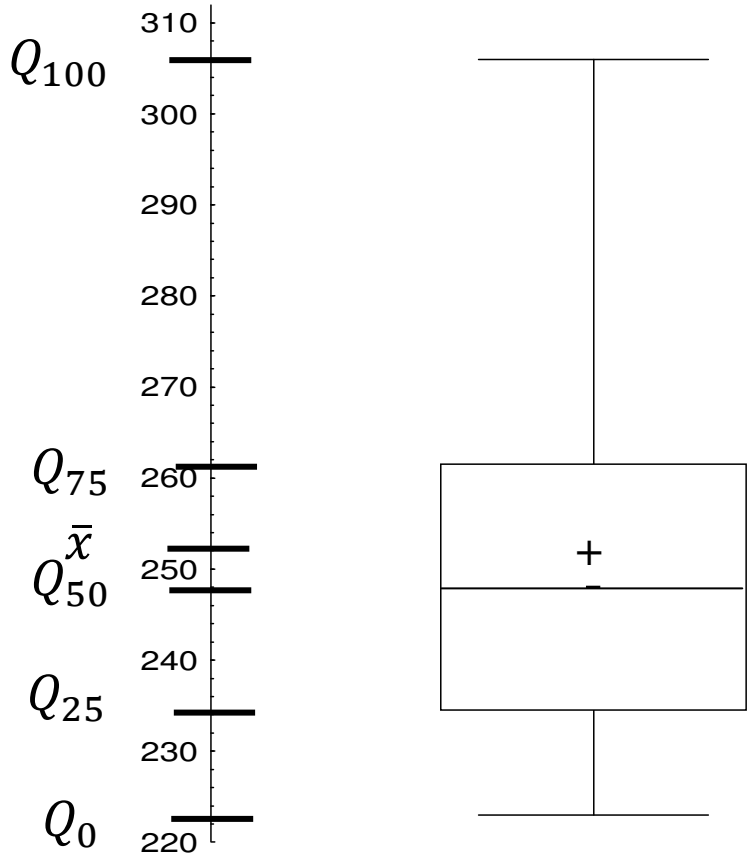
Range

$$V = Q_{100} - Q_0 = x_{max} - x_{min} = 306 - 223 = 83$$

Interquartile range

$$R_{IQ} = Q_{75} - Q_{25} = 261.5 - 234.5 = 27$$

Box-Plot



4. Given are the weights of cattle in a small herd in kg ($n = 13$ animals).

295 248 260 223 306 234 263 248 235 251 232 244 267

Calculate the Quantiles $Q_0, Q_{25}, Q_{50}, Q_{75}, Q_{100}$

Solution:

The ascending sequence of sorted values

223	232	234	235	244	248	248	251	260	263	267	295	306
$x_{[1]}$	$x_{[2]}$	$x_{[3]}$	$x_{[4]}$	$x_{[5]}$	$x_{[6]}$	$x_{[7]}$	$x_{[8]}$	$x_{[9]}$	$x_{[10]}$	$x_{[11]}$	$x_{[12]}$	$x_{[13]}$

$$n = 13$$

$Q_0 = 223$

$Q_{25}: t = 25;\ p = 0.25;\ n \cdot p = 13 \cdot 0.25 = 3.25;\ j = 3;\ g = 0.25;\ Q_{25} = x_{[4]} = 235$

$Q_{50}: t = 50;\ p = 0.50;\ n \cdot p = 13 \cdot 0.50 = 6.50;\ j = 3;\ g = 0.50;\ Q_{50} = x_{[7]} = 248$

$Q_{75}: t = 75;\ p = 0.75;\ n \cdot p = 13 \cdot 0.75 = 9.75;\ j = 9;\ g = 0.75;\ Q_{75} = x_{[10]} = 263$

$Q_{100} = 306$