Exercise Part 6 with Solutions

1. Calculate the following limits:

a)

$$\lim_{x\to 0}\frac{\sqrt{1-x}-1}{x}=\lim_{x\to 0}\frac{\sqrt{1-x}-1}{x}\frac{\sqrt{1-x}+1}{\sqrt{1-x}+1}=\lim_{x\to 0}\frac{\left(\sqrt{1-x}\right)^2-1^2}{x\left(\sqrt{1-x}+1\right)}$$

$$=\lim_{x\to 0}\frac{1-x-1}{x\left(\sqrt{1-x}+1\right)}=\lim_{x\to 0}\frac{-x}{x\left(\sqrt{1-x}+1\right)}=\lim_{x\to 0}\frac{-1}{\sqrt{1-x}+1}$$

$$=\lim_{x\to 0}\frac{-1}{\sqrt{1-0}+1}=-\frac{1}{2}$$

b)

$$\lim_{x\to 0}[cot5x \cdot sin2x] = \lim_{x\to 0}\left[\frac{cos5x}{sin5x} \cdot sin2x\right] = \frac{2}{5}\lim_{x\to 0}\left[\frac{cos5x}{sin5x} \cdot sin2x \cdot \frac{5x}{2x}\right] =$$

$$\frac{2}{5}\lim_{x\to 0}\left[\underbrace{cos5x}_{\to 1} \cdot \frac{5x}{\underbrace{sin5x}_{\to 1}} \cdot \frac{sin2x}{\underbrace{2x}_{\to 1}}\right] = \frac{2}{5}$$

2. Find derivatives of the following functions:

a)

$$f(x) = 4x^3 + 2x^2 - 5x + 3$$

$$f'(x) = 12x^2 + 4x - 5$$

b)

$$f(x) = \frac{(3x^2 - 1)}{x^3}$$

$$f'(x) = \frac{6x \cdot x^3 - (3x^2 - 1) \cdot 3x^2}{(x^3)^2} = \frac{-3x^2 + 3}{x^4}$$

c)

$$f(x) = 2x^2 + 3x + \frac{1}{x^4}$$

$$f'(x) = 4x + 3 - \frac{4}{x^5}$$

d)

$$f(x) = (4x + 3)^2(2x^2 - 2)$$

$$f'(x) = 2 \cdot (4x + 3) \cdot 4 \cdot (2x^2 - 2) + (4x + 3)^2 \cdot 4x = 4(32x^3 + 36x^2 - 7x - 12)$$

e)

$$f(x) = \frac{4x^3 - 4x}{(3x+3)^3}$$

$$f'(x) = \frac{(12x^2 - 4)(3x+3)^3 - (4x^3 - 4x) \cdot 3 \cdot 3(3x+3)^2}{((3x+3)^3)^2} =$$

$$= \frac{(12x^2 - 4)}{(3x+3)^3} - \frac{9 \cdot (4x^3 - 4x)}{(3x+3)^4}$$

f)

$$f(x) = ln\left(\frac{1+2x}{2x-1}\right) + x^2$$

$$f'(x) = \frac{2x-1}{1+2x} \cdot \frac{2 \cdot (2x-1) - (1+2x) \cdot 2}{(2x-1)^2} + 2x$$

g)

$$f(x) = 3cos(x) + \sin(3x)$$

$$f'(x) = -3sin(x) + 3cos(3x)$$

h)

$$f(x) = 4cos(x^2) \cdot sin(x)$$

$$f'(x) = 4 \cdot 2x \cdot ((-\sin(x^2)) \cdot sin(x) + 4cos(x^2) \cdot cos(x)$$

$$= -8xsin(x^2)sin(x) + 4cos(x^2)cos(x)$$

i)

$$f(x) = 2xsin(x)$$

$$f'(x) = 2sin(x) + 2xcos(x)$$

j)

$$f(x) = (x-3) \cdot e^{(x-2)}$$

$$f'(x) = 1 \cdot e^{(x-2)} + (x-3) \cdot 1 \cdot e^{x-2} = (x-2) \cdot e^{x-2}$$

k)

$$f(x) = 25x^2+4e^{(x-2)^2}$$

$$f'(x) = 2 \cdot 25x+4 \cdot 2(x-2) \cdot 1 \cdot e^{(x-2)^2} = 50x + (8x-16)e^{(x-2)^2}$$

l)

$$f(x) = 2log_{10}e^{0,5x}$$

$$f'(x) = 2 \cdot \frac{1}{e^{0,5x}ln10}e^{0,5x} \cdot 0,5 = \frac{1}{ln10}$$