Exercise Part 5 with Solutions

1. Write the first 5 terms of the following sequence:

$$\{a_n\} = \left\{\left(\frac{1}{5}\right)^{2n}\right\}$$

$$\{a_n\} = \left\{\left(\frac{1}{5}\right)^{2};\ \left(\frac{1}{5}\right)^{4};\ \left(\frac{1}{5}\right)^{6};\ \left(\frac{1}{5}\right)^{8};\ \left(\frac{1}{5}\right)^{10}\right\}$$

2.

Write the first 5 terms of the following sequence. Does the sequence converge or diverge?

$$a_1 = 2$$

$$a_n = \left(\frac{1}{2}\right)^n \cdot a_{n-1}$$

$$a_n = \left\{2; 2 \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{2}; \frac{1}{2} \cdot \left(\frac{1}{2}\right)^3 = \frac{1}{16}; \frac{1}{16} \cdot \left(\frac{1}{2}\right)^4 = \frac{1}{256}; \frac{1}{256} \cdot \left(\frac{1}{2}\right)^5 = \frac{1}{8192}\right\}$$

the sequence converges to 0.

3.

Write all summands of the following sums

a)

$$\sum_{i=1}^{5} a = 5 \cdot a$$

b)

$$\sum_{k=21}^{23} \frac{k}{k+2} = \frac{21}{21+2} + \frac{22}{22+2} + \frac{23}{23+2} = \frac{21}{23} + \frac{22}{24} + \frac{23}{25}$$

$$= \frac{21 \cdot 24 \cdot 25 + 22 \cdot 23 \cdot 25 + 23 \cdot 23 \cdot 24}{23 \cdot 24 \cdot 25} = \frac{12600 + 12650 + 12696}{13800} = 1.9525$$

c)

$$\sum_{j=1}^{4} a_j \cdot x^j = a_1 \cdot x^1 + a_2 \cdot x^2 + a_3 \cdot x^3 + a_4 \cdot x^4$$

4. Calculate the following sums

a)

$$\sum_{k=2}^{4}\sum_{j=1}^{5}(j+k) = \sum_{k=2}^{4}(1+k+2+k+3+k+4+k+5+k) = \sum_{k=2}^{4}(15+5k) =$$

$$15+5\cdot 2+15+5\cdot 3+15+5\cdot 4 = 15\cdot 3+5(2+3+4) = 45+45 = 90$$

b)

$$\left(\sum_{k=2}^{4} k\right)\left(\sum_{j=1}^{3} j\right) = (2+3+4)\cdot(1+2+3)$$

$$= 2\cdot 1 + 2\cdot 2 + 2\cdot 3 + 3\cdot 1 + 3\cdot 2 + 3\cdot 3 + 4\cdot 1 + 4\cdot 2 + 4\cdot 3$$

$$= 2+4+6+3+6+9+4+8+12 = 54$$

5. Given are the following measurements for y_{ij}:

j \ i	**1**	**2**	**3**	**4**	**5**
1	1	2	1	3	6
2	3	5	3	1	5
3	4	3	2	5	1
4	6	8	2	3	2

Calculate the following sums:

$$\sum_{i=1}^{3}\left(\sum_{j=1}^{2} y_{ij}\right)^2$$

Solution

	1	**2**	**3**	**4**	**5**
1	1	2	1	3	6
2	3	5	3	1	5
3	4	3	2	5	1
4	6	8	2	3	2

$$\sum_{i=1}^{3}\left(\sum_{j=1}^{2} y_{ij}\right)^2 = \sum_{i=1}^{3}(y_{i1} + y_{i2})^2 = (y_{11} + y_{12})^2 + (y_{21} + y_{22})^2 + (y_{31} + y_{32})^2$$

$$= (1 + 3)^2 + (2 + 5)^2 + (1 + 3)^2 = 16 + 49 + 16 = 81$$

6. Calculate:

a)

$$\frac{90!}{87!} = \frac{90 \cdot 89 \cdot 88 \cdot (87!)}{(87!)} = 90 \cdot 89 \cdot 88$$

b)

$$\sum_{i=2}^{4} i! = 2! + 3! + 4! = 2 + 6 + 24 = 32$$

7. Determine whether the following limit exists:

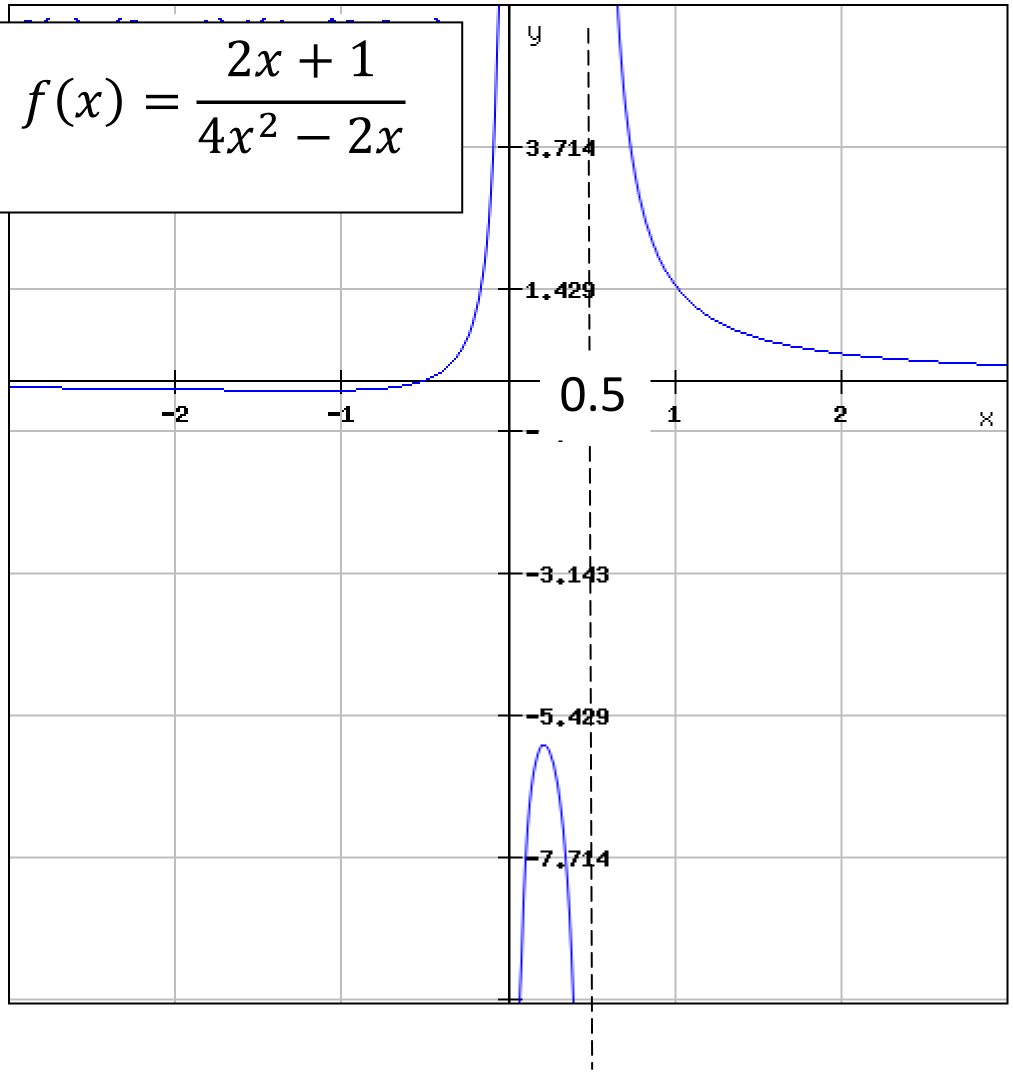
$$\lim_{x \to 0.5} \frac{2x+1}{4x^2-2x}$$

$$\lim_{x \to 0.5^+} \frac{2x+1}{4x^2-2x} = \frac{2+\frac{1}{x}}{4x-2} = \frac{2+\frac{1}{x \to 0.5^+}}{4(x \to 0.5^+)-2} = \frac{4^-}{0^+} = +\infty$$

$$\lim_{x \to 0.5^-} \frac{2x+1}{4x^2-2x} = \frac{2+\frac{1}{x}}{4x-2} = \frac{2+\frac{1}{x \to 0.5^-}}{4(x \to 0.5^-)-2} = \frac{4^+}{0^-} = -\infty$$

The limit does not exist.

$$f(x) = \frac{2x+1}{4x^2-2x}$$



8. Calculate for $x \to \infty$; $x \to -\infty$ and $x \to 0$ the limits of following functions:

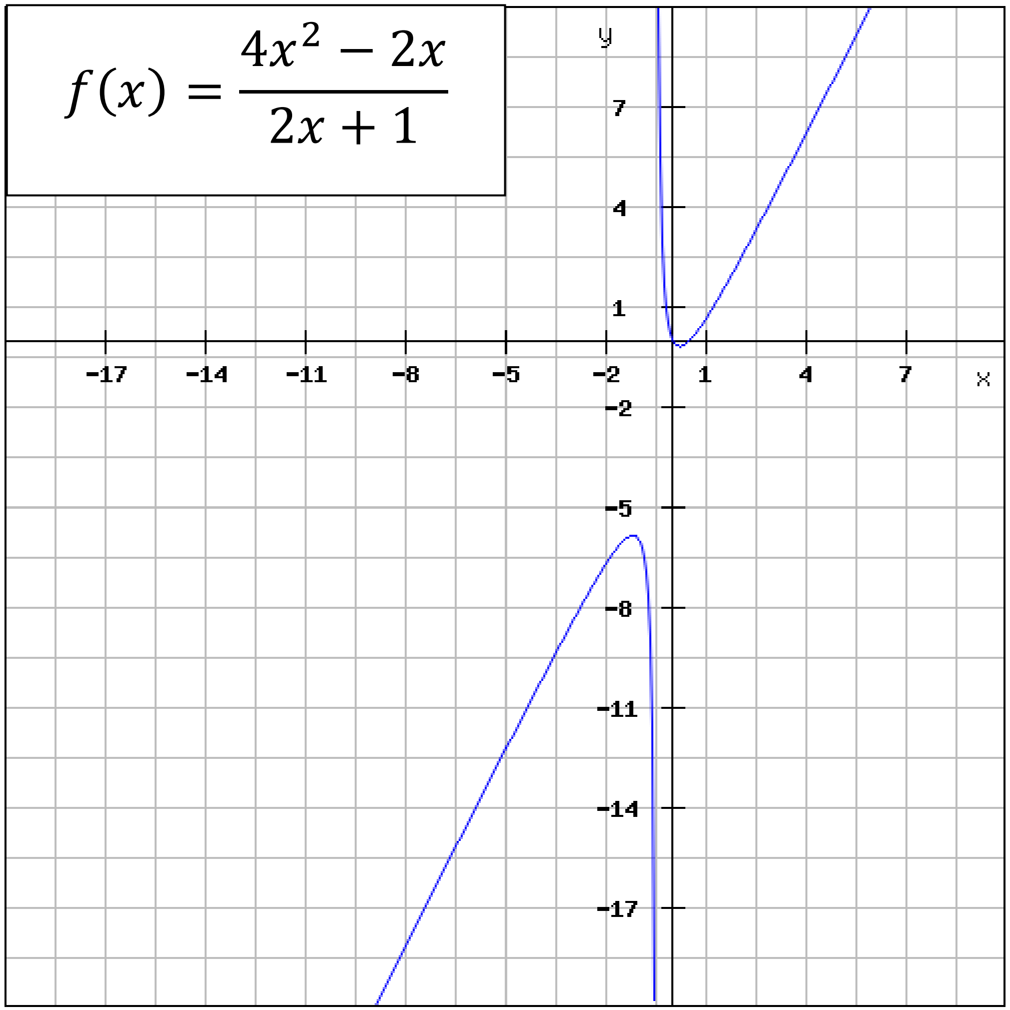
a)

$$f(x) = \frac{4x^2 - 2x}{2x + 1}$$

$$\lim_{x\to\infty} \frac{4x^2 - 2x}{2x + 1} = [Deg\ top > Deg\ bottom, l.c > 0] = \infty$$

$$\lim_{x\to-\infty} \frac{4x^2 - 2x}{2x + 1} = [Deg\ top > Deg\ bottom, l.c > 0] = -\infty$$

$$\lim_{x\to 0} \frac{4x^2 - 2x}{2x + 1} = 0$$



$f(x) = \frac{4x^2 - 2x}{2x+1}$
y
7
4
1
-2
-5
-8
-11
-14
-17
-17
-14
-11
-8
-5
-2
1
4
7
x

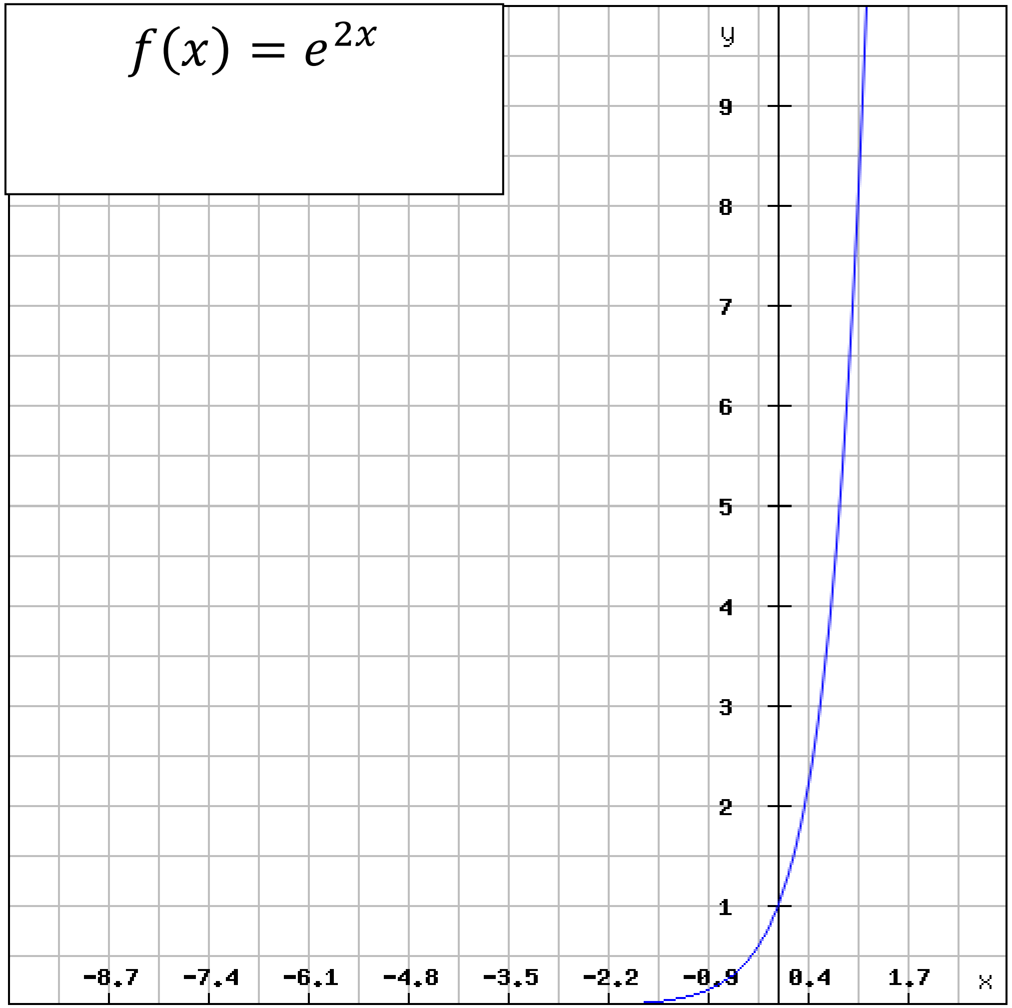
b)

$$f(x) = e^{2x}$$

$$\lim_{x \to \infty} e^{2x} = \infty$$

$$\lim_{x \to -\infty} e^{2x} = 0$$

$$\lim_{x \to 0} e^{2x} = 1$$



9. Calculate the following limits:

a)

$$\lim_{x \to \infty} [5x^3 - 2x^2 + 17x - 27] = \lim_{x \to \infty} [5x^3] = \infty$$

b)

$$\lim_{x \to \infty} \left[\frac{6x^2 - 3}{3x^2 + 5} \right] = \frac{l.c. of\ top}{l.c. of\ bottom} = \frac{6}{3} = 2$$

c)

$$\lim_{x\to\infty}\left[\frac{14x^2-4x+5}{4x^5-3x^3+7x^2-35}\right]=[Deg\ top < Deg\ bottom]=0$$

d)

$$\lim_{x\to-\infty}\left[\frac{-5x^3+6x^2-4}{2x^2-7x+2}\right]=[Deg\ top > Deg\ bottom, l.c\ of\ the\ top < 0]=+\infty$$