## Computer Science and Mathematics Summer term 2014

## Exercises 2

1. The following sets are sets of number pairs, i.e., subsets of the cartesian product  $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$ :

$$A := \{ (x, y) \in \mathbb{R}^2 \mid y = \frac{2}{3}x - 2 \}$$

$$B := \{ (x, y) \in \mathbb{R}^2 \mid y = -|0.5x| + 1 \}$$

$$B := \{ (x, y) \in \mathbb{R}^2 \mid y = -|0.5x| + 1 \}$$
  
$$C := \{ (x, y) \in \mathbb{R}^2 \mid x \ge 0 \land -x + 3 \ge y \ge 0 \}$$

- (a) Visualize each of the sets A, B, C in the cartesian coordinate system. (Make a separate graphical image for each set.)
- (b) All these sets are relations. Which of them are even functions?
- 2. Let P be the set of all participants of a party and D the set of all drinks which are offered there. Each participant  $p_i \in P$  shall get a drink  $d_i \in D$  according to his/her preference. In this way, a mapping  $f: P \rightarrow D$  shall be defined.

What do the following possible properties of f mean in this context?

- (a) surjectivity
- (c) bijectivity
- (b) injectivity
- (d) Can a participant of the party get two different drinks?
- 3. Let  $f: D \to R$  be the function described by  $f(x) = 2x^3 1$ .

The domain *D* is defined as  $D := \{ x \in \mathbb{R} \mid -3 \le x < 2 \}$ . Determine

- (a) the range R := f(D),
- (b) a formula for the inverse function  $f^{-1}$ .
- 4. (a) Calculate the decimal value of the binary number 1001111.
  - (b) Calculate the hexadecimal representation of the decimal number 999.
  - (c) What is the binary expansion of the value 1/3?

(Hint: You can do "written division" analogously to the decimal case, but with doubling the remainder in every step instead of multiplying by 10.)

- 5. Let  $\vec{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ .
  - (a) Draw  $\vec{a}$  and  $\vec{b}$  in a coordinate system.
  - (b) Determine  $2 \cdot \vec{a} + \vec{b}$  by calculation and graphically.
  - (c) Let  $\vec{c} = \begin{pmatrix} 1 \\ -11 \end{pmatrix}$ . Find a representation of  $\vec{c}$  as a linear combination of  $\vec{a}$  and  $\vec{b}$ .
- 6. Let  $\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ . What geometrical objects are described by the following sets?

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(a) { 
$$\vec{b} + t \cdot \vec{a} \mid t \in \mathbb{R} \land t \ge 0$$
 }

(b) { 
$$\vec{x} \in \mathbb{R}^2 \mid \vec{a} \cdot \vec{x} = 0$$
 }

(c) { 
$$\vec{x} \in \mathbb{R}^2 \mid ||\vec{x} - \vec{b}|| = 0.5$$
 }

- 7. (a) Are the vectors  $\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ ,  $\vec{b} = \begin{pmatrix} -3 \\ -2 \\ 5 \end{pmatrix}$  and  $\vec{c} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$  linearly independent?
  - (b) What is the maximal number of vectors which can be linearly independent in  $\mathbb{R}^4$ ?
- 8. The points A = (1; 3), B = (11; 7) and C = (3; 13) are given in the cartesian coordinate system.
  - (a) Let A be the new zero (origin) and calculate the vectors  $\vec{b} = \overrightarrow{AB}$  and  $\vec{c} = \overrightarrow{AC}$ .
  - (b) Calculate the vector  $\vec{d} = \vec{b} + \vec{c} = \overrightarrow{AD}$  and the absolute coordinates of the new point D.
  - (c) Calculate the inner product  $\vec{b} \cdot \vec{c}$  and the angle  $\angle(\vec{b}, \vec{c})$ .
  - (d) Extend the vectors by a third dimension (with value 0) and calculate the cross product  $\vec{b} \times \vec{c}$  .
  - (e) Calculate the area of the parallelogram spanned by  $\vec{b}$  and  $\vec{c}$ .