Computer Science and Mathematics
Summer term 2014

Exercises 1

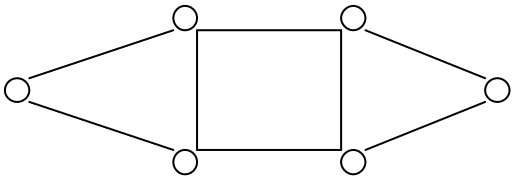
1. Show with Venn diagrams:
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (b) $(A \cap B)^C = A^C \cup B^C$

2. (a) How many subsets with exactly 3 elements does a set with 5 elements have?
   (b) Let $B(n, k)$ = number of k-element subsets of a set with n elements.
   Show that $B(n, k) = B(n–1, k) + B(n–1, k–1)$ if $n \geq 1$ and $0 < k \leq n$.
   (c) From (b), deduce the list of numbers $B(n, k)$ for $n = 0, 1, ..., 6$ and $k = 0, 1, ... n$.
   In which other mathematical context do they appear?

3. Find a formula for $|A \cup B \cup C|$.

4. What is the number of words of length n over an alphabet with k elements?
   List them systematically for the case $k = 2$, $n = 4$.

5. How many circles (with no edges appearing more than one time in it) are contained in this graph?



6. List all bipartite graphs with each of the two constituting vertex sets having 2 elements.

7. List all functions of the set { a; b; c } into itself.
   Which of them are bijective?

8. Which of the following functions $f: \mathbb{R} \to \mathbb{R}$ are bijective?
   $f(x) = 1$
   $f(x) = 2x$
   $f(x) = x^2$
   $f(x) = 2^x$
   $f(x) = x^3$

9. Determine a formula for the inverse function f^{-1} for $f(x) = 1/(2x+1)$ (with $x > -1/2$).

10. (just for training) Simplify as far as possible:
    (a) $\dfrac{1-\dfrac{x-1}{x+1}}{x-\dfrac{1}{x}}$
    (b) $(a + 2b)^2 – (b – 2a)^2 + (a – b)(a + b)$
    (c) $f(g(x))$ for $f(z) = z^3 + z + 1$ and $g(x) = x – 1$.