University of Göttingen
Department Ecoinformatics, Biometrics and Forest Growth
Dr. Irina Kuzyakova, Prof. Dr. Joachim Saborowski, Prof. Dr. Winfried Kurth

Computer Science and Mathematics, summer term 2013
Test exam

	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Σ
max.	7	13	10	10	5	5	7	3	60
credits									

Please, read all tasks carefully.
Write down the way how you got your result; credits are given also for the right approach towards a solution.

Time for this exam: 90 min.; no electronic devices are allowed.

Task 1. (Linear algebra: vectors)

Three vectors in $\mathbb{R}^3$ are given:

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}, \ \vec{c} = \begin{pmatrix} -16 \\ 0 \\ -10 \end{pmatrix}.$$

(a) Calculate the vector $2 \cdot \vec{a} + \vec{b}$. *(2 cr.)*

(b) Calculate the inner product $\vec{a} \cdot \vec{b}$. *(1 cr.)*

(c) What is the angle between $\vec{a}$ and $\vec{b}$? *(1 cr.)*

(d) Are the three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ linearly independent? Prove your answer. *(2 cr.)*

(e) Give a geometrical description of the shape of the set of points in space described by
$\{ \ \vec{x} \in \mathbb{R}^3 \mid \vec{c} \cdot \vec{x} = 0 \ \}$
("·" denotes the inner product of vectors.) *(1 cr.)*

Task 2. (Linear algebra: matrices)

The matrix $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is given.

(a) What is the rank of A ? Give a reason for your answer. *(1 cr.)*

(b) Calculate $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Draw the vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $A \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

in a cartesian coordinate system. *(4 cr.)*
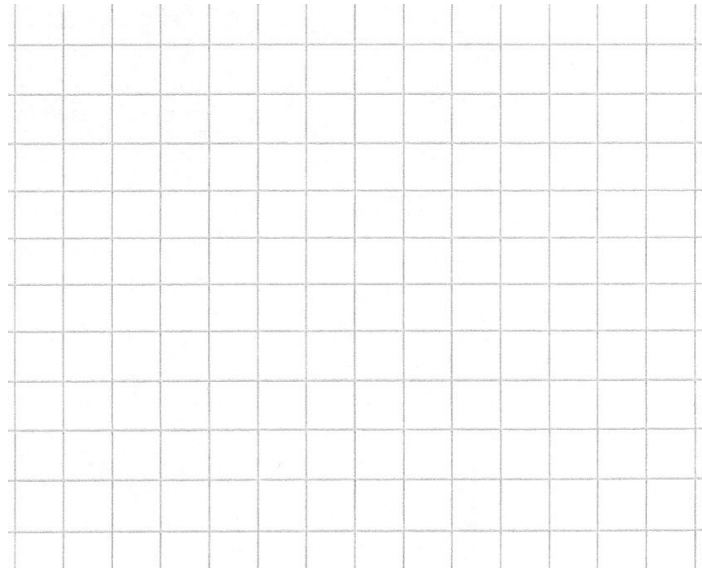
(c) The linear mapping associated with A is $f : \vec{x} \mapsto A \cdot \vec{x}$.
Describe (in words) how an arbitrary vector $\vec{x}$ is transformed geometrically by f.

(1 cr.)

(d) Calculate the matrix A^2. *(1 cr.)*

(e) Calculate det A. *(1 cr.)*

(f) Determine the matrix A^{-1} (if it exists). *(2 cr.)*

(g) Determine the eigenvalues of A. *(3 cr.)*

Task 3. (Calculus: univariate functions, differentiation) *(10 cr.)*

Given is the following function: $f(x) = \frac{1}{3}x^3 - 4x^2 + 7x - 5$.

(a) Find all x values where the function f has local extrema and classify them as minima or maxima.

(b) Find where the function is increasing / decreasing, and all x values of inflection points.

Task 4. (Calculus: integration) *(10 cr.)*

Compute the total area between the function $f(x) = 6x^2 + 6x - 12$, the x axis and the lines $x_1 = 0$ and $x_2 = 2$.

Task 5. (Computer science: programming) *(5 cr.)*

The following Java method **f** gets an integer array **x** and a single integer **a** as its arguments:

```
public int f(int x[], int a)
   {
   int i = 0;
   boolean b = true;
   while (b && (i < x.length))
      {
      if (x[i] == a)
         b = false;
      else
         i = i+1;
      }
   if (b)
      {
      println("Error!");
      return -1;
      }
   else
      return i;
   }
```

What does it give back as its result?

Task 6. (Computer science: representation of numbers)

(a) What is the binary representation of the decimal number 63 ?   *(1 cr.)*

(b) What is the decimal representation of the hexadecimal number 2A5 ?   *(1 cr.)*

(c) Give the 8-bit two's complement of the decimal number –84.   *(2 cr.)*

(d) What is the exact value of the fraction which is represented in the ternary system (= base 3) by 0.2222222... (with infinitely many 2s after the dot) ?   *(1 cr.)*

Task 7. (Statistics: confidence interval and one-sided test)   *(7 cr.)*

On a farm the weight of 100 cows was measured. The mean body weight was $\bar{x} = 720$ kg and the variance of the sample was $s_x^2 = 400$ kg. The body weight is approximately normal distributed.

(a) Compute a 95 % confidence interval for the mean weight.

(b) Formulate the null hypothesis and answer the question if the measured average weight of the 100 cows is significantly higher than 700 kg (α=0,05).

Task 8. (Statistics: linear regression, correlation)   *(3 cr.)*

Investigated was the relationship between the *height* of apple trees in m (*x*)
and the *yield* in kg (*y*).

The estimated regression equation was as follows:
$\widehat{y} = -0.5 + 1.4x$,

and the estimate of the coefficient of correlation $r = 0.96$.

(a) What yield do you expect by the height = 3 m ?

(b) What fraction of the variability of *y* can be explained by *x* ? Give your answer in words.