Exercises 1

1. Show with Venn diagrams:
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (b) $(A \cap B)^C = A^C \cup B^C$

2. (a) How many subsets with exactly 3 elements does a set with 5 elements have?

   (b) Let $B(n, k)$ = number of k-element subsets of a set with n elements.
      Show that $B(n, k) = B(n-1, k) + B(n-1, k-1)$ if $n \geq 1$ and $0 < k \leq n$.

   (c) From (b), deduce the list of numbers $B(n, k)$ for $n = 0, 1, ..., 6$ and $k = 0, 1, ...\ n$.
      In which other mathematical context do they appear?

3. Find a formula for $|A \cup B \cup C|$.

4. What is the number of words of length n over an alphabet with k elements?
   List them systematically for the case $k = 2,\ n = 4$.