Exercise 8

a) Find the indefinite integral for following functions.

a)

$$f(x) = (2x+1)(4x^3+2) + cos\,(x)$$

$$f(x) = 8x^4 + 4x^3 + 4x + 2 + cos\,(x)$$

$$F(x) = \frac{8}{5}x^5 + x^4 + 2x^2 + 2x + sinx + C$$

b)

$$f(x) = e^{2x+1} + \frac{4}{2x+3}$$

$$Substitution: \begin{bmatrix} t = 2x+1; & \frac{dt}{dx} = 2; dx = \frac{dt}{2} \\ s = 2x+3; & \frac{ds}{dx} = 2; dx = \frac{ds}{2} \end{bmatrix}$$

$$F(x) = \int e^{2x+1}dx + \int \frac{4}{2x+3}dx = \int \frac{1}{2}e^t dt + \int \frac{1}{2}\cdot\frac{4}{s}ds =$$
$$= \frac{1}{2}e^t + 2\,ln\,s = \frac{1}{2}e^{2x+1} + 2\,ln(2x+3) + C$$

c)

$$f(x) = \left(x + \frac{1}{3}\right) sin\ x$$

Integration by Parts:

$$\int u'(x) \cdot v(x)dx = uv - \int u(x) \cdot v'(x)$$

$$\begin{bmatrix} u'(x) = sinx\ ; u(x) = -cosx \\ v(x) = \left(x + \frac{1}{3}\right);\ v'(x) = 1 \end{bmatrix}$$

$$F(x) = \int sinx \cdot \left(x + \frac{1}{3}\right) dx = -cosx \cdot \left(x + \frac{1}{3}\right) - \int (-cosx \cdot 1)dx =$$
$$= -cosx \cdot \left(x + \frac{1}{3}\right) - (-sinx) = sinx - cosx\left(x + \frac{1}{3}\right) + C$$

2. Find the definite integral for following functions.

a)

$$f(x) = 2^{2x+1}$$

1. The Indefinite Integration by Substitution

$$Substitution: \left[t = 2x + 1;\ \frac{dt}{dx} = 2; dx = \frac{dt}{2}\right]$$

$$\int 2^{2x+1}dx = \int \frac{1}{2} 2^t\, dt = \frac{1}{2ln2} 2^t = \frac{1}{2ln2} 2^{2x+1}$$

$$\int_1^3 2^{2x+1}\, dx = \left[\frac{1}{2ln2} 2^{2x+1}\right]_1^3 = \frac{1}{2ln2}(2^7 - 2^3) = \frac{1}{ln2} \cdot 60 \approx 86{,}56$$

2. The definite Integration by Substitution

$$\begin{bmatrix} t = 2x + 1 \\ t(1) = 3 \\ t(3) = 7 \end{bmatrix}$$

$$\int_1^3 2^{2x+1}\, dx = \left[\frac{1}{2ln2} 2^t\right]_3^7 = \frac{1}{2ln2}(2^7 - 2^3) \approx 86{,}56$$

b)

$$f(x) = 3x^2 \cdot e^x$$

Integration by Parts:

$$\int u'(x) \cdot v(x)dx = uv - \int u(x) \cdot v'(x)$$

$$\begin{bmatrix} u'(x) = e^x ; u(x) = e^x \\ v(x) = 3x^2;\ v'(x) = 6x \end{bmatrix}$$

$$\int 3x^2 \cdot e^x\, dx = e^x \cdot 3x^2 - \int e^x \cdot 6x\, dx = e^x \cdot 3x^2 - (e^x \cdot 6x - e^x \cdot 6) =$$
$$= 3e^x(x^2 - 2x + 2)$$

$$\int_{-4}^{-1} 3x^2 \cdot e^x\, dx = [3e^x(x^2 - 2x + 2)]_{-4}^{-1} =$$
$$= 3e^{(-1)}((-1)^2 - 2(-1) + 2) - 3e^{(-4)}((-4)^2 - 2(-4) + 2)) \approx 4{,}09$$

c)

$$f(x) = 4x^3 + xcos(5x^2)$$

The definite Integration by Substitution

$$\begin{bmatrix} t(x) = 5x^2;\ \frac{dt}{dx} = 10x;\ dx = \frac{dt}{10x} \\ t(1) = 5 \cdot 1^2 = 5 \\ t(3) = 5 \cdot 3^2 = 45 \end{bmatrix}$$

$$\int_1^3 (4x^3 + xcos(5x^2)\, dx = \int_1^3 4x^3 + \int_5^{45} \frac{1}{10} cost\, dt$$

$$= \int_1^3 4x^3 + \int_5^{45} \frac{1}{10} sint\, dt =$$

$$\left[4 \cdot \frac{1}{4} \cdot x^4\right]_1^3 + \left[\frac{1}{10} sint\right]_5^{45} = 3^4 - 3^1 + \frac{1}{10} sin45 - \frac{1}{10} sin5 \approx 79{,}81$$

3. Compute the total area between the follow functions and the x-axis and the lines x_1 and x_2

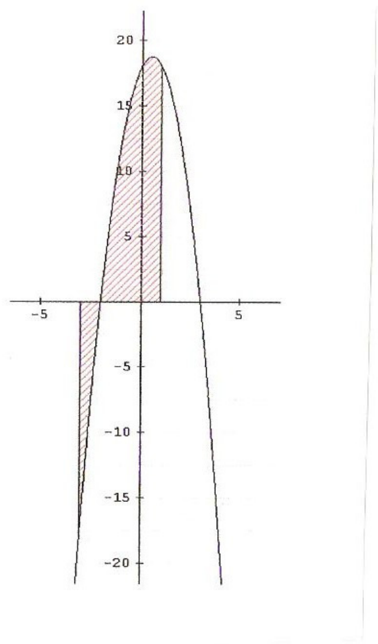
a)

$$f(x) = -3x^2 + 3x + 18$$

Between $x_1 = -3 \; and \; x_2 = 1$

The roots of the function:

$$-3x^2 + 3x + 18 = 3(-x^2 + x + 6) = 0$$
$$-x^2 + x + 6 = 0$$
$$x_{1,2} = \frac{-1 \pm \sqrt{25}}{-2}$$
$$x_1 = -2; \; x_2 = 3$$



$$Area = \left| \int_{-3}^{-2} (-3x^2 + 3x + 18) dx \right| + \int_{-2}^{1} (-3x^2 + 3x + 18) dx =$$

$$|[-x^3 + 1{,}5x^2 + 18x]_{-3}^{-2}| + [-x^3 + 1{,}5x^2 + 18x]_{-2}^{1} =$$

$$= |8 + 6 - 36 - (27 + 13{,}5 - 54))| - 1 + 1{,}5 + 18$$
$$- (8 + 6 - 36) = 8{,}5 + 40{,}5 = 49$$

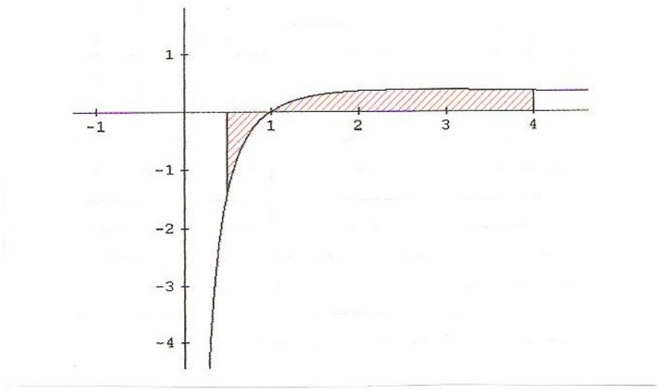
b)

$$f(x) = \frac{1}{x} lnx$$

Between $x_1 = 0{,}5 \; and \; x_2 = 4$

The roots of the function:

$$\frac{1}{x} lnx = 0; lnx = 0 \Longrightarrow x = 1$$



The definite Integration by Substitution

$$\left[\begin{array}{c} t(x) = lnx \\ dx = xdt \\ \int tdt = \frac{t^2}{2} = \frac{(lnx)^2}{2} \end{array}\right]$$

$$Area = \left|\int_{0,5}^{1} \frac{1}{x} lnxdx\right| + \int_{1}^{4} \frac{1}{x} lnx\,dx = \left|\left[\frac{(lnx)^2}{2}\right]_{0,5}^{1}\right| + \left[\frac{(lnx)^2}{2}\right]_{1}^{4} =$$

$$= \left|\frac{1}{2}(\ln(1))^2 - \frac{1}{2}(\ln(0{,}5))^2\right| + \frac{1}{2}(\ln(4))^2 - \frac{1}{2}(\ln(1))^2$$

$$= \left|0 - \frac{1}{2}(\ln(0{,}5))^2\right| + \frac{1}{2}(\ln(4))^2 - 0 \approx 1{,}20$$

c)

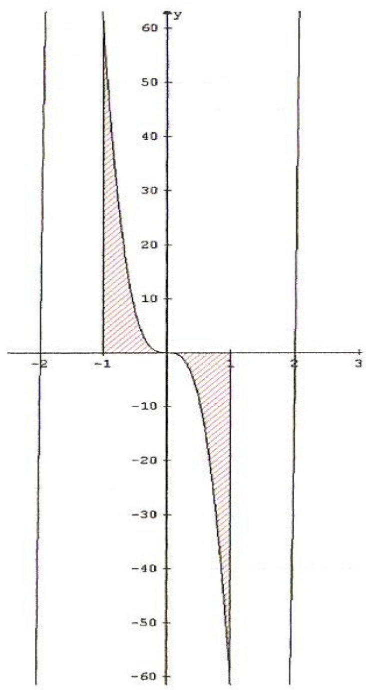
$$f(x) = 4x^3(x^4 - 16)$$

Between $x_1 = -1 \; and \; x_2 = 1$

The roots of the function:

$$4x^3(x^4 - 16) = 0$$

$$x_1 = -2; x_2 = 0; \; x_3 = 2$$



$$Area = \int_{-1}^{0} 4x^3(x^4 - 16)dx + \left| \int_{0}^{1} 4x^3(x^4 - 16)\,dx \right|$$

$$= \left[\frac{1}{2}x^8 - 16x^4\right]_{-1}^{0} + \left| \left[\frac{1}{2}x^8 - 16x^4\right]_{0}^{1} \right| =$$

$$= 0 - 0 - \left(\frac{1}{2} - 16\right) + \left| \left(\frac{1}{2} - 16\right) - (0 - 0) \right| = 31$$

4. Compute the total area between the follow curves $f(x)$ and $g(x)$:

a)

$$f(x) = 2x + 1; \quad g(x) = (x-1)^2$$

Intersections:

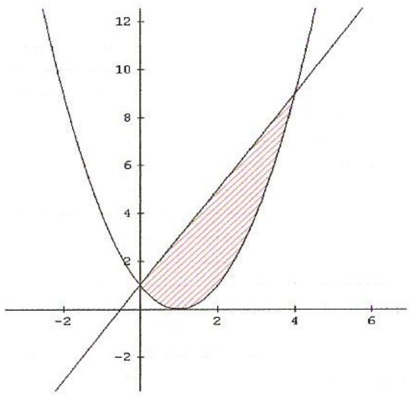
$$2x + 1 = (x-1)^2$$

$$2x + 1 = x^2 - 2x + 1$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x_1 = 0;\ x_2 = 4$$



$$f(x) = 2x + 1: ceiling;\ g(x) = (x-1)^2: floor$$

$$Area = \int_0^4 (2x+1)dx - \int_0^4 ((x-1)^2)dx = \int_0^4 (2x + 1 - (x-1)^2)dx =$$

$$= \int_0^4 \big(2x + 1 - (x^2 - 2x + 1)\big)dx = \int_0^4 (-x^2 + 4x)dx = \left[-\frac{1}{3}x^3 + 2x^2\right]_0^4 =$$

$$= -\frac{1}{3} \cdot 4^3 + 2 \cdot 4^2 + 0 - 0 = 10\frac{2}{3}$$

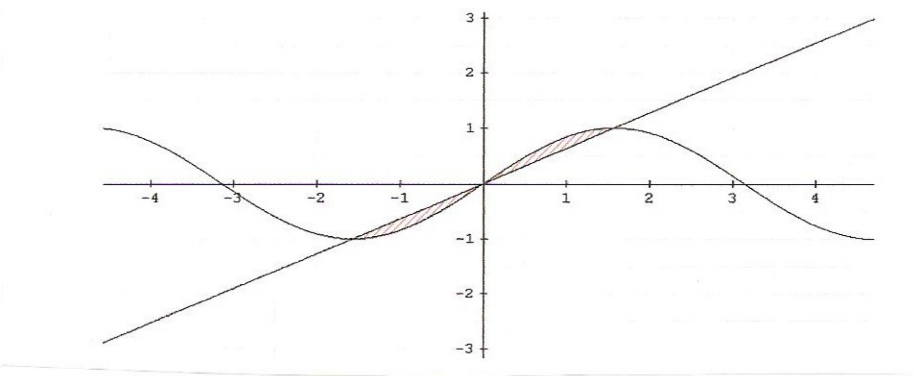
b)

$$f(x) = \frac{2}{\pi}x; \quad g(x) = sinx$$

Intersections:

$$\frac{2}{\pi}x = sinx$$

$$x_1 = -\frac{\pi}{2};\ x_2 = 0;\ x_3 = \frac{\pi}{2}$$



$$g(x) = sinx: ceiling;\ f(x) = \frac{2}{\pi}x: floor$$

$$Area = \left|\int_{-\frac{\pi}{2}}^{0}\left(sinx - \frac{2}{\pi}x\right)dx\right| + \int_{0}^{\frac{\pi}{2}}\left(sinx - \frac{2}{\pi}x\right)dx$$

$$= \left|\left[-cosx - \frac{1}{\pi}x^2\right]_{-\frac{\pi}{2}}^{0}\right| + \left[-cosx - \frac{1}{\pi}x^2\right]_{0}^{\frac{\pi}{2}}$$

$$= \left|-1 - 0 - \left(0 - \frac{\pi}{4}\right)\right| + 0 - \frac{\pi}{4} - (-1 - 0) = 2 - \frac{\pi}{2} \approx 0{,}43$$